SME Policy Reform in the Networked Economy: Reallocation and Network Destruction*

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Abstract

This paper studies the dynamic welfare implications of cutting subsidies for Small and Medium Enterprises (SMEs) in an economy where firms are interdependent through production networks. The subsidy policy reform reallocates resources from inefficient small firms to efficient large firms by expelling small firms from the economy and can improve welfare in the long run. In the short run, however, the exit of small firms destroys production networks and damages large firms' production in the networked economy. To study the two countering effects on welfare, we develop a general equilibrium model with firms' exit decisions and an endogenous formation of firm-level production networks. Using the model, we first characterize how the small firms' exit destructs production networks and damages other firms in the networked economy. Next, using calibrated models, we quantitatively show that the network destruction effect surpasses the reallocation effect in the short run, and the economy falls into a temporary recession just after the policy reform. In the long run, the networks are reconstructed, and welfare is improved through the reallocation of labor.

Keywords: Production Networks, Shock Propagation, Network Destruction, Misallocation, Industrial Policy

JEL Classification: D21, D24, D57, D85, E22, E23, E61

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1 Introduction

The literature on misallocation has pointed out that preferential policies for SMEs distort resource allocation and lead to aggregate welfare losses. This is because inefficient SMEs will use production resources too much, leaving fewer resources available for more efficient large firms. Japan is a particularly SME-friendly country, with 40% of SMEs supported by government credit subsidy policies (OECD, 2016). Some of those firms could have exited without these policies. Research in the misallocation literature indicates that, in the long run, reforming these policies will result in significant welfare improvements through reallocating resources from those inefficient firms to more efficient firms. However, it is unclear whether such policies also improve welfare in the short run. While much of the research on misallocation has discussed policy effects by comparing a steady state with a wedge causing misallocation and one without it, little has been said about the negative impacts that may occur before reaching a better steady state in the short run.

In the short run, one substantial side effect of the policy reform emerges if we consider production networks between firms that are not immediately re-constructed. The exiting firms may have functioned as suppliers or customers to the remaining firms, and their sudden exit has the direct negative externality of reducing productivity of the surviving firms as fewer suppliers or reducing sales as fewer customers. Furthermore, these shocks could be transmitted to indirect suppliers and customers through the production network among surviving firms. Suppose these side effects are large enough to cancel out the gains from labor reallocation to efficient large firms from exiting small firms. In that case, even if it is a welfare-improving policy in the long run where production networks are reconstructed, there will be a significant negative shock to the economy in the short run due to the destruction of production networks.

In this paper, we develop a general equilibrium (GE) model of endogenous network formation/destruction and reallocation with heterogeneous firms to investigate the implication of policy reform in the economy with a firm-to-firm production network. Our model builds on the firm-to-firm network formation literature using matching function expressions like Lim (2018) but extends these models by incorporating endogenous exit and reallocation. These margins are critical to assess the effects of network destruction due to customer/supplier death in the short run and the reallocation of resources from exiting small firms to larger firms by the policy reform.

The model is static and consists of firms with constant elasticity of substitution (CES) production functions. They make decisions in the order of construction of the production network, stay/exit subject to random operating costs, and production. First, an entrant constructs a network comparing the expected profit gained via the network and the network cost randomly given; for the supplier, networking will increase the mass of customers and increase the revenue, and for the customer, networking will increase productivity due to the love of variety effect of the CES production function. Next, firms receive a random operation cost and make a stay/exit decision comparing the variable profit gained by production and the operating cost. The operation cost can be covered by a subsidy so that even a firm whose profit is smaller than the operation cost can participate in the production. Finally, firms produce and sell goods using production networks created at the beginning of the period.

Using this model, we first show analytically that the external effects of the sudden exit of small firms due to the subsidy policy reform on surviving firms through the disruption of production networks can be decomposed into *direct/indirect network effects* and in the *upstream/downstream* direction through networks. Suppose an unanticipated reform takes place after the network has been constructed and forces small firms that cannot cover their operational costs without subsidies to exit. Then, the surviving firms simultaneously lose suppliers and customers. Firms that lose suppliers will experience a decrease in productivity due to the loss of variety effect of the CES production function, which we denote as the downstream direct network effect. Furthermore, the increase in the price of the firm's intermediate goods due to the lowered productivity, in turn, decreases its customer's productivity and continues to propagate downstream via the production networks, which we denote the downstream indirect network effect. Firms that lose customers will experience a decrease in sales and this, in turn, leads to a decrease in intermediate demand on its suppliers and continue to propagate upstream via the network, which we denote upstream direct/indirect network effect in the same way. In this paper, we characterize the way of downstream (upstream) propagation by a simple expression using a discounted sum of a *supplier (customer) network matrix* which simply denotes the mass of suppliers (customers) of each firm. We link the expression to two essential concepts in other fields. The first is the *Leontief inverse* in the field of inter-industry relationships, and the second is *Bonacich centrality* in the field of social network analysis.

Next, we simulate the subsidy policy reform in two calibrated economies, one in which small firms function mainly as customers on the network and the other in which they function mainly as suppliers, to evaluate how the network effects and reallocation effects propagate via the production networks. In both economies, the effects of network destruction outweigh the reallocation effect in the short run, resulting in a decrease in welfare, but the economies archive higher welfare after the network is reconstructed accommodating the policy change, due to reallocation. However, a significant difference emerges in the driving forces in the short-run welfare decline for the two economies. In an economy where small firms function as customers, the downstream network effect caused by customer deaths leads to chain failures, increasing the CPI due to the loss of variety effect. However, in an economy where small firms function as suppliers, the short-run decline in welfare is mainly driven by the decrease in productivity of large firms due to the upstream network effect caused by supplier deaths.

Related Literature

This paper is linked to different strands of literature. First, it is related to the literature on the propagation of shocks through production networks.[1] Beginning with Leontief (1941), this literature has long studied propagation via the linkages between industries based on input-output tables both empirically (e.g., Long Jr and Plosser, 1983; Acemoglu et al., 2016) and theoretically (e.g., Leontief, 1941; Hulten, 1978; Acemoglu et al., 2012; Baqaee and Farhi, 2018, 2019). Recent empirical studies reveal propagation between firms via customer-supplier networks in the use of firm-level production network data (e.g., Barrot and Sauvagnat, 2016; Boehm et al., 2019; Carvalho et al., 2021; Miyauchi, 2021). The corresponding theory, however, is scant, unlike propagation between industries. This is because when considering shock propagation between firms, in addition to the intensive margin of productivity or demand shocks discussed in between-industry propagation, one must also consider the extensive margin by firms' exit that destroys the network itself. Recent studies like Taschereau-Dumouchel (2022), Acemoglu and Tahbaz-Salehi (2020), and Elliott et al. (2022) tackle these problems in fashionable ways,[2] but they describe the destruction of *discrete links*,

[1] For surveys on this literature, check Carvalho and Tahbaz-Salehi (2019) for an extensive summary of propagation both between industries and firms, Bernard and Moxnes (2018) and Elliott and Golub (2022) for recent topics about propagation between firms.

[2] Taschereau-Dumouchel (2022) considers an economy with a finite number of suppliers/customers and propose a new solution technique to solve nonconvex optimization problems with binary variables like firms' exit and network

which complicates the models and makes it hard to derive analytical expressions of propagation or incorporate other structures like misallocation. This study, however, takes a totally different approach from them and characterizes the network as a product of a *matching function* and the mass of firms like Lim (2018). This enables us to express the destruction of the network due to firms' exit as if it is an intensive margin, i.e., a decrease in the mass of firms, given the matching function constant. We utilize this separation and derive propositions on how the network destruction due to firms' exit propagates to the entire economy and relate the results to *Leontief inverse matrix* and *Bonacich centrality*, which are widely used in the field of input-output analysis and social networks analysis, respectively.

Second, this paper is related to the recently growing literature on endogenous production network formation in general equilibrium (e.g., Oberfield, 2018; Eaton et al., 2022; Acemoglu and Azar, 2020; Lim, 2018; Huneeus, 2020; Bernard et al., 2022; Tintelnot et al., 2018; Dhyne et al., 2021; Kopytov et al., 2022; Elliott et al., 2022). The network formation decision in this paper is based on costly-relationship network formation used in Lim (2018), Huneeus (2020), and Bernard et al. (2022). We mainly extended the model of Lim (2018) in the direction of including the entry and exit of firms, representing the destruction of networks by an unanticipated exit of firms in addition to the construction. This paper also contributes to the literature by showing the importance of identification on the source of firms' heterogeneity in the endogenously-networked economy. By assuming two different assumptions on the source of primitive heterogeneity, we construct two calibrated models which yield similar firm size distributions but different production network structures due to the endogenous network formation. We show that the two models behave quite differently in response to shocks due to the difference in the structures of production network.

Finally, this paper is related to the vast literature on misallocation. Starting from the quantification of its significance by Hsieh and Klenow (2009), various channels have been discussed as a source of misallocation.[3] Our paper focuses on the misallocation of labor resources to small, inefficient firms which would have exited without a wedge. Such firms have been particularly mentioned in the studies of "zombie firms" named by Caballero et al. (2008), which points out that about 10% of Japanese firms are insolvent borrowers but remain to exist thanks to a subsidized loan.[4] In the context of misallocation literature, Kwon et al. (2015) and Hosono and Takizawa (2022) empirically show misallocation of labor inputs to the zombie firms decreased the aggregate TFP of Japan in the 1990s.[5] This paper contributes to the literature by pointing out the short-run side

formation using *relaxed problems*. The solution technique is strong and attractive, but analytical expressions are hard to obtain in the model. Acemoglu and Tahbaz-Salehi (2020) considers a similar economy with strategic network formation using Rubinstein bargaining. While they proved some important features like the existence and uniqueness of the subgame perfect equilibrium, quantitative implications are hard to derive. Elliott et al. (2022) adopted a novel approach assuming continuous investment choice that determines the probability of each discrete link being active, which avoids the computational intractability of discrete networks. It derives various interesting phenomena like *phase transition* with discrete complex network structures, but hard to obtain welfare implications or analytical expressions. In fact, the economy is not in GE and excludes firms' heterogeneity.

[3]For an extensive survey of various channels, check Hopenhayn (2014) and Restuccia and Rogerson (2017).

[4]Strictly speaking, the original work of Caballero et al. (2008)'s attribution of the reason for the existence of these zombie firms to the actions of the banks is different from our assumption of a government wedge. Imai (2016), however, points out that the government credit subsidy has led to an increase in zombie firms. In recent years, considering the side effects of the government policies for the COVID-19 pandemic, more studies are pointing out that these policies increasing zombie firms. For a survey of empirical results from different countries on the expansion of zombie firms by these policies, see Yamada et al. (2022).

[5]About misallocation of other resources by a similar wedge, Liu (2019) empirically shows government subsidies on zombie firms in China misallocate capital to these firms, and Acemoglu et al. (2018) show subsidy for operating cost can help such firms survive and misallocate resources of R&D to them using a structurally estimated model and

effects of eliminating these wedges due to the destruction of production networks. While recent papers in this field tend to point out the possible short-run negative impact of policy changes using transition dynamics (e.g., Atkeson et al., 2019; Alessandria et al., 2021; Edmond et al., 2018)[6], as far as we know, this is the first attempt to analyze the negative impact in the short run via the network destruction. To archive this purpose, we analytically show the existence of the short-run side effects first. Second, we quantitatively show the short-run damage to the economy due to the network destruction is significant enough to offset reallocation effects, and the welfare temporarily declines. In the long run, however, welfare becomes certainly larger than its original level before the reform through reallocation and network reconstruction.[7]

Outline

The outline of this paper is as follows. Section 2 develops a general equilibrium model with an exogenously given network structure. Section 5 analytically presents how the subsidy policy change and resulting exit of firms propagate via a firm-to-firm network in the exogenous network equilibrium defined in 2. Section 4 extends the model constructed in Section 2 by endogenizing network formation to relate the structure of the network and firm characteristics. Section 5 quantitatively presents the dynamic effect of policy changes using calibrated GE models developed in Section 4. Section 6 concludes.

2 Exogenous Network Model

In this section, we introduce a GE model with an exogenous network structure and how to characterize the equilibrium. This helps to separately analyze the effects of the propagation of exit shock and network reconstruction.

2.1 Environment

The economy consists of a representative household and an endogenously determined continuum of firms producing a differentiated good. Firms are owned by the household, and all the profits are distributed to the household. Firms are heterogeneous over states $\chi = (\phi, \delta)$, where ϕ is *fundamental productivity* in a firm's production function and *fundamental demand* in the household utility function. The firm distribution over the state space S_χ is expressed as a cumulative distribution function $M(\chi)$. We assume all the subsidies are financed by a lump-sum tax on the household and repaid from firms to the household as a profit so that the subsidy does not affect the household behavior. For brevity, we refer to a firm with state χ as χ-firm.

its simulation.

[6] Literature that treats household-side heterogeneity has studied the differences in policy evaluation over several time horizons for a long time. For example, Lucas Jr (1990) shows reducing capital tax improves welfare in the long run, but the potential welfare gain is notably smaller once transition dynamics are taken into account.

[7] There is literature that studies static propagation of wedge in the steady state of an economy with production networks like Jones (2011), Grassi et al. (2017), Bigio and La'o (2020), Baqaee and Farhi (2020) and Caliendo et al. (2022). While both our research and the literature analyze an inefficient economy with production networks, the literature analyzes how micro distortions, such as monopoly power or idiosyncratic tax, spread throughout the economy through the network in steady state, unlike dynamic propagation as ours.

2.1.1 Households

The representative household supplies L units of general labor inelastically every period and has CES preferences below over all goods in the economy. The preference is given by

$$U = \left[\int [\delta x^H(\chi)]^{\frac{\sigma-1}{\sigma}} dM(\chi) \right]^{\frac{\sigma}{\sigma-1}}. \tag{2.1}$$

Here, σ is the elasticity of substitution across varieties, and $x^H(\chi)$ is the household's consumption of χ-firm goods (as a final good). Subsidy on firms are financed by lump-sum tax, and it does not alter household behavior. Given price $p^H(\chi)$, household demand is written by

$$\begin{aligned} x^H(\chi) &= \left(P^H\right)^\sigma \left(p^H(\chi)\right)^{-\sigma} U\delta^{\sigma-1} && (2.2)\\ &= \Delta^H \delta^{\sigma-1} \left(p^H(\chi)\right)^{-\sigma}. && (2.3) \end{aligned}$$

Here, P^H is the consumer price index (CPI) given by

$$P^H = \left[\int \left(p^H(\chi)\right)^{1-\sigma} \delta^{\sigma-1} dM(\chi) \right]^{\frac{1}{1-\sigma}} \tag{2.4}$$

and we denote $\Delta^H = U(P^H)^\sigma$ as an aggregate demand shifter which determines the scale of demand on $x^H(\chi)$ and is related only to the aggregate variables, not to the individual firm characteristics χ.

2.1.2 Firm

Each χ-firm produces its differentiated output using labor and intermediate goods imported from other χ'-firms if there is a supplier-customer relationship between the customer χ and the supplier χ'. Following Lim (2018), we characterize this as a matching function $m(\chi, \chi')$ which determines a fraction that χ-firm can purchase inputs from a χ'-firm. For example, suppose there are 100 firms with firm character χ'. If $m(\chi, \chi') = 0.5$, a χ-customer has customer-supplier relationships with 50 χ'-suppliers, and the remaining 50 χ'-firms are not supplier for the χ-firm.

Given the expression on the network above, the production function of a χ-firm is given by

$$X(\chi) = \left[[\phi l(\chi)]^{1-\frac{1}{\sigma}} + \int \left(x(\chi, \chi')\right)^{1-\frac{1}{\sigma}} m(\chi, \chi') dM(\chi') \right]^{\frac{\sigma}{\sigma-1}} \tag{2.5}$$

where $l(\chi)$ is the quantity of labor input and $x(\chi, \chi')$ is the quantity of inputs from χ'-firm as intermediate goods.

Taking wage as numeraire and given the price $p(\chi, \chi')$ which is charged by χ'-supplier to χ-customer, the cost minimization problem of χ-customer implies marginal cost is given by:

$$\eta(\chi) = \left[\phi^{\sigma-1} + \int \left(p(\chi, \chi')\right)^{1-\sigma} m(\chi, \chi') dM(\chi') \right]^{\frac{1}{1-\sigma}} \tag{2.6}$$

and demand on each input, labor and intermediate goods are given by

$$\begin{aligned} l(\chi) &= X(\chi)\eta^\sigma(\chi)\phi^{\sigma-1} && (2.7)\\ x(\chi, \chi') &= \left(\eta(\chi)\right)^\sigma \left(p(\chi, \chi')\right)^{-\sigma} X(\chi). && (2.8) \end{aligned}$$

2.1.3 Market structure

The market structure is monopolistic competition. Since the final goods demand of household (2.3) and the intermediate goods demand of firms (2.8) imply they have the same price elasticity σ and it is constant, the profit maximization problem given the monopolistic competition structure implies

$$p^H(\chi) = p(\chi', \chi) = \mu\eta(\chi) \tag{2.9}$$

and $\mu = \dfrac{\sigma}{1-\sigma}$ is a markup rate which is constant across firms. Given the constant markup, the operating profit of χ-firm is written by

$$\pi(\chi) = (\mu - 1)\eta(\chi)X(\chi). \tag{2.10}$$

2.2 Entry and Exit

Figure 1 summarizes the timing of a firm's action. First, entry occurs at the beginning of the period, and fundamental characteristics and *operation cost* are realized. Operation cost is determined independently of fundamental characteristics, as we see more detail later. Next, each of the entrants constructs firm-to-firm relationships, which determine the economy's production network. This is exogenously given in this section while we endogenize in Section 4. Next, they make a stay/exit decision considering their profit by production and operation cost. This determines the distribution function of firms ($M(\chi)$) that produce in the period. After production, all the firms are liquidated at the end of the period.

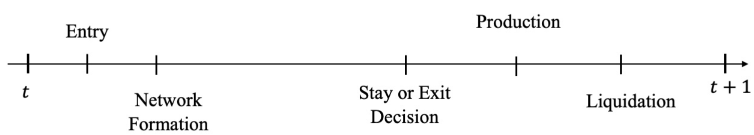


Figure 1: Timeline of firms' decision-making

2.2.1 Exit

A χ-potential firm endogenously exits before production if the profit obtained by production is smaller than the operation cost. We introduce random operating cost $\epsilon \underset{i.i.d.}{\sim} G_\epsilon(\epsilon)$.[8] in units of labor. Then, given an cot for network formation $c^f(\chi)$ which is endogenized in Section 4, we define χ-firm's value after realization of ϵ is defined as

$$V(\chi|\epsilon) = \max\{0, \pi^*(\chi) + s - \epsilon\} \tag{2.11}$$

[8] If its probability density function is Dirac's delta like $\delta(x) = 0$ ($x \neq c_\epsilon$) and $\delta(x) = \infty$ ($x = c_\epsilon$), this model is reduced to deterministic fixed operation cost c_ϵ like Melitz (2003). Random operation cost is also used in König et al. (2022).

where $\pi^*(\chi) = \pi(\chi) - c^f(\chi)$ is the profit that remains after network costs are subtracted, and s is a subsidy by the government. For simplicity, we assume subsidy is paid to all the firms in the economy.[9]

Additionally, we define endogenous survival rate $h(\chi)$, which determines the probability that χ-potential firm can survive operation cost realization and produce goods.

$$h(\chi) = Pr\left(\epsilon \leq \pi^*(\chi) + s\right) \tag{2.12}$$

$$= G_\epsilon\left(\pi^*(\chi) + s\right) \tag{2.13}$$

Note that since $G_\epsilon(\cdot)$ is a cumulative distribution function, this is increasing in profit. This implies potential firms with high profits are more likely to survive the period than ones with low profit. Lastly, we define the associated operation cost of the firms that survive at the period as

$$l^o(\chi) = c_\epsilon\left(\pi^*(\chi) + s\right) \tag{2.14}$$

where $c_\epsilon(x)$ denotes expected value of ϵ conditional on $\epsilon < x$ and expressed as

$$c_\epsilon\left(x\right) = \int_{-\infty}^{x} \epsilon \frac{dG_\epsilon(\epsilon)}{G_\epsilon(x)}. \tag{2.15}$$

2.2.2 Entry

The mass of entrants M^e is exogenously given. Firms' fundamental characteristics realize after entry and drawn from $\chi \underset{\text{i.i.d.}}{\sim} G_\chi(\chi)$.

2.2.3 Distribution of Operating Firms

From the law of large numbers, the probability that χ-potential firm survives (and produces goods) corresponds to a fraction of χ-potential firms that survive the realization of operation costs. Then, a distribution function about firms that actually produce outputs emerges as below.

$$M(\chi) = h(\chi) M^e G_\chi(\chi) \tag{2.16}$$

2.3 Market Clearing

Goods market clearing for the output of a χ-firm is given by:

$$X(\chi) = x^H(\chi) + \int x(\chi', \chi) m(\chi', \chi) dM(\chi'). \tag{2.17}$$

The first term on the right-hand side is a demand by the household as final goods, and the second term is an aggregation of demand by other firms as intermediate goods.

Labor market clearing is given by:

$$L = L^p + L^o \tag{2.18}$$

[9] Given a distribution $G_\epsilon(\epsilon)$ with enough small variance, this subsidy does not affect large firms because they can survive without it.

where

$$L^p = \int l(\chi) dM(\chi) \tag{2.19}$$

$$L^o = \int l^o(\chi) dM(\chi). \tag{2.20}$$

2.4 Network Expression

Here, to disentangle the complicated structure of the production network and separately analyze the effects of suppliers and customers, we define two important variables by change of variables. Using the two variables, we also introduce two fixed-point equations. The equations capture how the production networks determine firms' characteristics in combination with firms' fundamental characteristics.

2.4.1 Backward and Forward Fixed Point Equation

First, we define *productivity* $\Phi(\chi)$ as $(\eta(\chi))^{1-\sigma}$. Note that this is an inverse measure of a marginal cost. Raising marginal cost equation (2.6) to the power of $1-\sigma$, *backward fixed point equation* of the productivity measures[10] can be written as

$$\Phi(\chi) = \phi^{\sigma-1} + \beta_\Phi \int \Phi(\chi') m(\chi, \chi') dM(\chi') \tag{2.21}$$

where $\beta_\Phi = \mu^{1-\sigma}$.

Next, we define *scaled demand* $\Delta(\chi)$ as $\frac{1}{\Delta^H} X(\chi)\eta(\chi)^\sigma$. Note that this demand measure is (i) price-adjusted in the sense that $\eta(\chi)^\sigma$ is multiplied (remember the price elasticity of demand takes common value σ between household and firms) (ii) adjusted by the size of the economy in the sense that it is divided by Δ^H, which captures GE effect on demand as defined in (2.3) and not depend on firm characteristics χ. Thus, this measure captures the demand on χ-firm excluding its price effects and general equilibrium effects. Substituting household demand (2.3) and firms' demand (2.8) to goods market clearing condition (2.17), *forward fixed point equation* of the scaled demand can be written as

$$\Delta(\chi) = \mu^{-\sigma}\delta^{\sigma-1} + \beta_\Delta \int \Delta(\chi') m(\chi', \chi) dM(\chi') \tag{2.22}$$

where $\beta_\Delta = \mu^{-\sigma}$.

Figure 2 and figure 3 express how productivity and scaled demand are determined in the economy for the special case when there is no heterogeneity among firms. In this case, (2.22) and (2.21) can be simply expressed as

$$\Phi = \phi^{\sigma-1} + \beta_\Phi \Phi m M \tag{2.23}$$

$$\Delta = \mu^{-\sigma}\delta^{\sigma-1} + \beta_\Delta \Delta m M \tag{2.24}$$

[10] We follow the naming of Bernard et al. (2022) for similar determination of equilibrium prices and quantities over production networks.

respectively. First, let's focus on the determination of productivity using figure 2. The first term on the right-hand side captures the firm's productivity without its supplier network. In this networked economy, χ-firm's productivity is enhanced by utilizing the suppliers' intermediate goods captured by the second term. Furthermore, the gain in productivity is transmitted downstream to its customers, and it continues to the fixed point shown in Figure 2. Next, let's focus on the determination of scaled demand using figure 3. Remember that the forward fixed point equation is derived from the good market clearing condition. The first term with fundamental quality δ comes from household demand, and the second term is derived from the demand of other firms as an intermediate good. In the same logic with the determination of productivity, the demand on a firm transmits upstream to its suppliers, and scaled demand is determined as a fixed point shown in Figure 3.

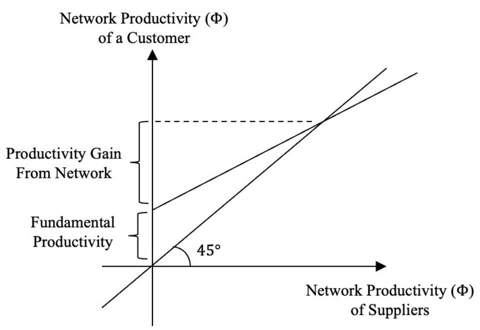


Figure 2: Productivity determination

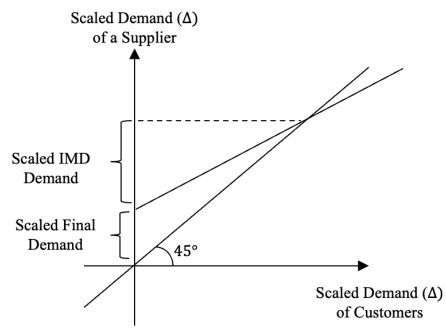


Figure 3: Scaled demand determination

2.4.2 Uniqueness of the Fixed Points

The expression on (2.21) and (2.22) allow us to confirm its uniqueness of productivity $\Phi(\chi)$ and scaled demand $\Delta(\chi)$. Hereafter, we assume the mass of entrants $M_e < \dfrac{1}{\beta_\Phi}$ holds.[11] Then, since $m(\chi, \chi'), h(\chi) \leq 1$ from the definition of probability, Blackwell's sufficient condition is satisfied for $\Phi(\chi)$ and $\Delta(\chi)$. Then, (2.21) and (2.22) constitute two contraction mappings. Hence, (2.21) and (2.22) have a unique solution from the contraction mapping theorem. Furthermore, the contraction mapping theorem implies iteration of the mapping defined in the right-hand side both in (2.21) and (2.22) guarantees convergence on the unique solution from any functions. We utilize the characters to quantitatively compute its equilibrium as shown in A.1.

[11] Since $\beta_\Phi = \mu^{1-\sigma}$ with $\sigma > 1$ and $\mu = \dfrac{\sigma}{\sigma - 1} > 1$, this assumption holds for standard value value of M_e.

2.4.3 Network Expression of Other Key Variables

Using productivity $\Phi(\chi)$ and scaled demand $\Delta(\chi)$, we rewrite associated variables of χ-firm as follows.

$$\pi(\chi) = (\mu - 1)\Delta^H \Delta(\chi)\Phi(\chi) \tag{2.25}$$

$$l(\chi) = \Delta^H \Delta(\chi)\phi^{\sigma-1} \tag{2.26}$$

The remaining is to determine aggregate demand shifter $\Delta^H = U(P^H)^\sigma$. Labor demand (2.26) and labor market clearing (2.18) implies

$$\Delta^H = \frac{L - L^o}{\int \Delta(\chi)\phi^{\sigma-1} dM(\chi)}. \tag{2.27}$$

The welfare is calculated by

$$U = \Delta^H (P^H)^{-\sigma} \tag{2.28}$$

given the CPI

$$P^H = \mu \left[\int \delta^{\sigma-1} \Phi(\chi) dM(\chi) \right]^{\frac{1}{1-\sigma}}. \tag{2.29}$$

2.5 Equilibrium

Finally, we define a general equilibrium given a production network structure. An equilibrium with exogenous networks is defined as follows.

Definition 1 (exogenous network equilibrium)**.** Given a matching function $m(\chi, \chi')$, network formation cost $c^f(\chi)$, and a subsidy policy s, an exogenous network equilibrium is allocation functions $\Delta(\chi), \Delta^H$, a price function $\Phi(\chi)$, a distribution function $M(\chi)$, such that (i) consumer chooses consumption to maximizes utility, (ii) potential firms make a stay/exit decision to maximize its value, (iii) firms choose production input to maximize its profit and (iv) all markets clear.

Note that the uniqueness of $\Phi(\chi)$ and $\Delta(\chi)$ discussed in 2.4.2 implies the uniqueness of the equilibrium.

3 Analytical Exercise

In this section, to clarify how small firms' exit due to policy change propagates through the production network to the entire economy, we analyze the general equilibrium model developed in Section 2. To concentrate on the propagation of firms' exit shock through the production network, we assume that the network is not reconstructed after the policy change in this section. This corresponds to the following timeline in figure 4 where the shocks occur after network formation. Hence, the effect of the policy change can be analyzed by comparative statics with changes in subsidy s using an exogenous network equilibrium defined in 1.[12]

[12] Kopytov et al. (2022) also adopts a model with this kind of timeline in the sense that some shock occurs after the network is formed. In their model, the network is formulated before productivity realization, and they analyze a relationship between the volatility of productivity shock and network formation.

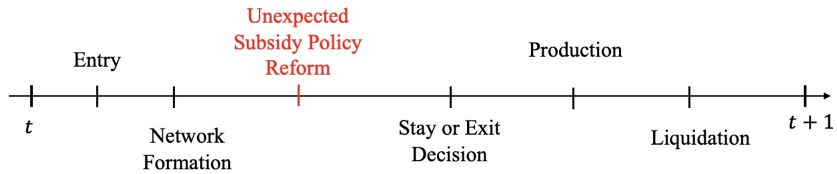
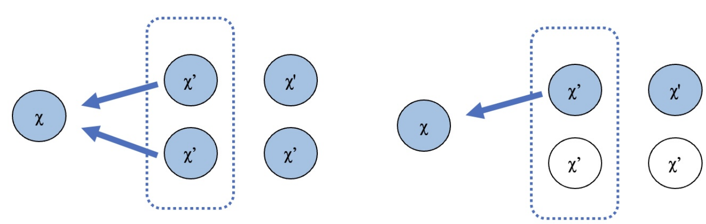


Figure 4: Timeline of firms' decision-making and policy change

3.1 Network Destruction

Here, we explain how networks are destructed after an abrupt policy reform. In both Figure 5a and Figure 5b, each of the vertices corresponds to one firm. Filled vertices are operating firms that survive operation cost shock with a realization of low operation cost $\epsilon \leq \pi(\chi)$, and blank vertices are non-operating (exiting) firms that cannot survive operation cost shock with a realization of low operation cost $\epsilon > \pi(\chi)$. Figure 5a shows the case of $h(\chi') = 1$ with a subsidy policy and Figure 5b shows $h(\chi') = 0.5$ after its reform.

Vertices surrounded by a dotted line are χ'-suppliers for a χ-customer, so the figure shows the case of $m(\chi, \chi') = 0.5$. A directed edge from one vertex to another implies the former supplies to the latter. As Figure 4 shows, firms cannot adjust networks just after the policy reform. Thus, the matching function is fixed. Since the number of the realized network is the product of matching probability and mass of firms, the destruction of networks exactly corresponds to the decline in the number of firms with a matching function fixed. The same discussion holds for customer death by reversing the direction of the edges.



(a) Without reform (b) After reform

Figure 5: Exit and Realized Network

Each vertex corresponds to one firm. Filled vertices are operating firms that survive operation cost shock. Blank vertices are non-operating (exiting) firms that cannot survive operation cost shock. Vertices surrounded by a dotted line are χ'-suppliers for a χ-customer. A directed edge from one vertex to another implies the former supplies to the latter. Figure 5a corresponds to $h(\chi') = 1$ with subsidy policy and Figure 5b corresponds to $h(\chi') = 0.5$ after subsidy policy reform. Both of the figures correspond to the situation with $m(\chi, \chi') = 0.5$.

3.2 Discretization

Throughout this section, we discretize state space S_χ defined in Section 2.[13] This helps our understanding in two ways. Firstly, this helps us to get an intuition about the structure of the production network and the way of propagation departing from the expression using matching functions. Secondly, the way of propagation in this setting can be characterized in a very familiar way with two essential concepts in the field of input-output analysis and social network analysis, respectively.

We express a generic function $x(\omega)$ from a discrete domain $\Omega = (\omega_1, \cdots, \omega_n)$ as $\boldsymbol{x} = \begin{pmatrix} x(\omega_1) \\ \vdots \\ x(\omega_n) \end{pmatrix}$, and $y(\omega, \omega')$ as $\boldsymbol{y} = \begin{pmatrix} y(\omega_1, \omega_1) & \cdots & y(\omega_1, \omega_n) \\ \vdots & \ddots & \vdots \\ y(\omega_n, \omega_1) & \cdots & y(\omega_n, \omega_n) \end{pmatrix}$.

Using the discrete state space expression above, we define a *supplier network matrix* and *customer network matrix* to make the following expressions simple.[14]

$$\boldsymbol{N_s} = \boldsymbol{m} \circ \boldsymbol{M'} = \begin{pmatrix} m_{11}M_1 & \cdots & m_{1n}M_n \\ \vdots & \ddots & \vdots \\ m_{n1}M_1 & \cdots & m_{nn}M_n \end{pmatrix} \tag{3.1}$$

$$\boldsymbol{N_c} = \boldsymbol{m'} \circ \boldsymbol{M'} = \begin{pmatrix} m_{11}M_1 & \cdots & m_{n1}M_n \\ \vdots & \ddots & \vdots \\ m_{1n}M_1 & \cdots & m_{nn}M_n \end{pmatrix} \tag{3.2}$$

The two definitions above are straightforward. $\boldsymbol{N_s}_{ij} = m_{ij}M_j$ is the product of matching probability between i-supplier and j-customer, and mass of j-firms. From the low of large numbers, this yields i-firm's mass of j-firm supplier. $\boldsymbol{N_c}_{ij}$ determines i-firm's mass of j-firm customer in the same way.[15]

3.3 Upstream and Downstream Propagation of Exit Shock

We now show two propositions that characterize the effect of firms' exit and its downstream/upstream propagation.

[13] Note that this discretization does not change any discussion so far because we had not utilized any character of continuity of the state space. In fact, totally the same discussion in this section can be done even in a continuous state space, using some concepts so-called *Neuman series* and *Fredholm integral equation of the second kind*. Each maps the concepts about the inverse of a matrix and linear equations in a discrete space to counterparts generalized in a continuous state space, respectively. Check Yosida (1980) for more detail about functional analysis.

[14] $\circ$ is an operator for Hadamard product, which returns element-wise product like $\boldsymbol{x} \circ \boldsymbol{z} = (x_1 z_1, \cdots, x_n z_n)'$ and $\boldsymbol{x} \circ \boldsymbol{y} = \begin{pmatrix} x_1 y_{11} & \cdots & x_1 y_{1n} \\ \vdots & \ddots & \vdots \\ x_n y_{n1} & \cdots & x_n y_{nn} \end{pmatrix}$.

[15] This is a similar but different concept to an *adjacency matrix*, which is used in network analysis. Though each (i, j)-element of an adjacency matrix is 0 or 1, which determines whether node i and j are connected, each element of the network matrix here determines i-firm's mass of supplier firms with j-type. This difference results in one difference in the transposition of the two matrices. In the adjacency matrix, the transposition of one matrix immediately returns the opposite relationship. However, this does not hold in the network matrix because the mass of suppliers is not necessarily the same if the masses of firms differ. As you can see from (3.1) and (3.2), In line with this observation, it can be immediately shown that $\boldsymbol{N_s} \circ \boldsymbol{M} = \boldsymbol{N_c}' \circ \boldsymbol{M}$ holds which corresponds to a transposition of the adjacency matrix.

Proposition 1 (Downstream propagation equation)**.** Suppose some policy reform occurs and the mass of firms changes by $d\boldsymbol{M}$. Then, associated changes in productivity $\boldsymbol{\Phi}$ by the network destruction and downstream propagation can be expressed as follows.

$$d\boldsymbol{\Phi} = \left(\underbrace{I}_{\text{Direct Network Effect}} + \underbrace{\beta_\Phi \boldsymbol{N_s} + \beta_\Phi^2 \boldsymbol{N_s}^2 + \cdots}_{\text{Indirect Network Effect}} \right) \beta_\Phi \boldsymbol{N_s} \left(\boldsymbol{\Phi} \circ \frac{d\boldsymbol{M}}{\boldsymbol{M}} \right) \tag{3.3}$$

$$= \boldsymbol{R_s} \left(\boldsymbol{\Phi} \circ \frac{d\boldsymbol{M}}{\boldsymbol{M}} \right) \tag{3.4}$$

where $\boldsymbol{R_s} = \sum_{k=1} (\beta_\Phi \boldsymbol{N_s})^k$ is *downstream influence matrix.*

Proposition 2 (Upstream propagation equation)**.** Suppose some policy reform occurs and the mass of firms changes by $d\boldsymbol{M}$. Then, associated changes in scaled demand $\boldsymbol{\Delta}$ by the network destruction and upstream propagation can be expressed as follows.

$$d\boldsymbol{\Delta} = \left(\underbrace{I}_{\text{Direct Network Effect}} + \underbrace{\beta_\Delta \boldsymbol{N_c} + \beta_\Delta^2 \boldsymbol{N_c}^2 + \cdots}_{\text{Indirect Network Effect}} \right) \beta_\Delta \boldsymbol{N_c} \left(\boldsymbol{\Delta} \circ \frac{d\boldsymbol{M}}{\boldsymbol{M}} \right) \tag{3.5}$$

$$= \boldsymbol{R_c} \left(\boldsymbol{\Delta} \circ \frac{d\boldsymbol{M}}{\boldsymbol{M}} \right) \tag{3.6}$$

where $\boldsymbol{R_c} = \sum_{k=1} (\beta_\Delta \boldsymbol{N_c})^k$ is *upstream influence matrix.*

The propositions are worth mentioning in three senses. Firstly, they disentangle the two complex effects: the destruction of production networks through the exit of firms and its further propagation through the remaining network structure. The first factor is the direct network effect arising from firms' direct supplier/customer death and expressed in the first term in (3.3) and (3.5). In the downstream propagation, once a firm's supplier dies, the productivity of the firm decreases due to the loss of variety in the production function. Also, in the upstream propagation equation, once a firm's customer dies, the demand for the firm's intermediate goods declines. They are the direct effects of firms' exit, which destructs production networks. In this networked economy, however, the shock propagates further via remaining production networks. Once the firm's productivity decreases, this affects its customer's productivity via the higher pricing of the firm on its intermediate goods. This is captured by the second term, and this iterated propagation is expressed in each term after the second term. Also, in the upstream propagation equation, a decline in the firm's demand decreases its demand on intermediate inputs, and this indirect effect propagates upstream via the production networks. Examples and graphical interpretations of these direct/indirect network effects are also discussed in example 3.3.2.1.[16]

[16] The convergence rates of this propagation is determined by $\beta_\Phi = \mu^{1-\sigma}$ and $\beta_\Delta = \mu^{-\sigma}$. They increase in markup μ because high markup implies less profitability of using intermediate goods comparing labor. This decreases the economy's dependence on intermediate goods. They also increase in elasticity of substitution σ because firms more

Secondly, these expressions allow us to relate the effect of network destruction to two major concepts in the field of input-output analysis and the field of social network analysis. In the first field, the literature of input-output analysis, Leontief (1941) invented *Leontief inverse matrix* which is defined as $\boldsymbol{L} = \sum_{k=0} \boldsymbol{A}^k$ given the input-output matrix $\boldsymbol{A}$ in the economy.[17] In the economy with Ccobb-Douglas production and utility, Leontief inverse matrix represents the direct/indirect impact of a shock in one industry on other industries through inter-industry linkages. The influence matrix here has a similar expression as Leontief about the direct/indirect impact of the exit of $i-$firms and associated network destruction through inter-firm linkages. In this sense, the influence matrix extends the concept of the Leontief inverse matrix to inter-firm networks[18] and represents the shock of the destruction of networks due to firms' exit.[19] In the second field, the social network analysis, a measure so-called *weighted Bonacich centrality* proposed by Bonacich (1987)[20] corresponds to the expression in (3.4) and (3.6). This measure is calculated as a product of *Bonacich matrix* and primitive influence vector $\boldsymbol{\alpha}$. Given an adjacency matrix $\boldsymbol{N}$, the Bonacich matrix is the sum of the number of direct/indirect links between two nodes defined as $\sum_{k=1} (\beta \boldsymbol{N})^k$ where $\beta < 1$ captures the importance of networks. This measure captures the impact that a node is receiving from the entire network through direct or indirect relationships between other nodes with primitive influences $\boldsymbol{\alpha}$. Utilizing the network matrix expression above in (3.4) and (3.6), we can relate the effects of exits of firms that destructs the network itself to the concept that captures nodes' importance over networks. Furthermore, we can conclude that the downstream/upstream effect of i-firms' exit on j-firm becomes larger when (1) the number of direct or indirect connections as a customer/supplier in the sense of the Bonacich matrix is larger, (2) the level of productivity/scaled demand of j-firm is higher, (3) the decline in its mass is larger, all of which are very intuitive.

Finally, the influence matrix $\boldsymbol{R}$ is observable in the network structure given a single constant scalar β without depending on the information about the productivity or amounts of production of each firm. Thus, given the real network data, we can easily construct the matrix and forecast how the small firms' exit propagates to the economy via the production network up to the first-order effect.[21] We conduct this analysis in section 5 and relate the matrix information, which indicates the first-order effect and resulting propagation which indicates the infinite order effect in simulation and check if the analysis based on the matrix is informative.

severely evaluate increases in the cost of intermediate goods due to the positive markup. This decreases the economy's dependence on intermediate goods, too. Note that since this model assumes monopolistic competition on pricing, an increase in σ implies a decrease in μ. This implies an increase in the structural parameter σ has two countering effects on β. In Appendix D, we prove an increase in σ implies a decrease in β_Φ and an increase in β_Δ.

[17] A wide discussion about Leontief inverse can be seen in a survey by Carvalho and Tahbaz-Salehi (2019).

[18] In Carvalho et al. (2021), similar expressions are obtained about the propagation of intensive margin of capital-augmenting productivity shock using discrete firm-to-firm production networks. One stark difference is that the influence matrix enables us to analyze the extensive margin of network destruction due to firms' exit.

[19] Baqaee (2018), and Baqaee and Farhi (2020) analyze the inter-industry propagation under firms' entry/exit in each industry and extend the Leontief inverse matrix. In the models, firms certainly exit given some shock, but not directly damaging their customers and suppliers. Customers and suppliers for the exiting firms can immediately find alternative firms in their models.

[20] In the original paper Bonacich (1987), the primitive influence vector $\boldsymbol{\alpha}$ is $\mathbf{1}$, which implies no heterogeneity of nodes except for the network structure. Heterogeneous influences are introduced by Ballester et al. (2006) in the context of a network game. In the context of macroeconomics, Acemoglu et al. (2012) argues the effect of productivity shocks in a sector with high Bonacich centrality on aggregate output using an input-output matrix.

[21] Note that $\boldsymbol{R}$ can tell us which firm the exit of small firms will affect, but not to what extent. We need $\boldsymbol{\Phi}$ and $\boldsymbol{\Delta}$ of small firms to evaluate the quantitative level.

3.3.1 Effects on Real Value

Here, we map changes in productivity $\mathbf{\Phi}$ and scaled demand $\mathbf{\Delta}$ to changes in real economic variables. From (2.25), we can write the change in profit as

$$\frac{d\boldsymbol{\pi}}{\boldsymbol{\pi}} = \frac{d\Delta^H}{\Delta^H} + \frac{d\mathbf{\Delta}}{\mathbf{\Delta}} + \frac{d\mathbf{\Phi}}{\mathbf{\Phi}}. \tag{3.7}$$

This implies that changes in profit of i-firm can be decomposed into three components: the GE effect, which is determined independently by firm characteristics, the change in scaled demand, which is determined by its downstream network structure and direct/indirect customer death, and the change in productivity, which is determined by its upstream network structure and direct/indirect supplier death.

Given the change in profit $d\boldsymbol{\pi}$, from (2.16), we can write the change in mass as

$$\frac{d\boldsymbol{M}}{\boldsymbol{M}} = \boldsymbol{\theta} \circ \left(\frac{d\boldsymbol{\pi}}{\boldsymbol{\pi}^*} + \frac{ds}{\boldsymbol{\pi}^*} \right) \tag{3.8}$$

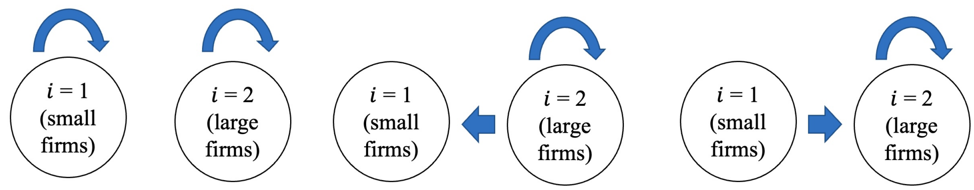
where $\theta_i = \left. \frac{\partial G_\epsilon / G_\epsilon}{\partial \epsilon / \epsilon} \right|_{\epsilon = \pi_i^*}$ is an elasticity between mass of firms and profit.[22] Note that given a probability density function of operation cost with upper-bounded support $-\infty < x < a$, for a certain large i-firm with $\pi_i^* > a$, $\theta_i = g_\epsilon(\pi_i^*)\pi_i^* = 0$ holds.[23] This implies all the firms large enough to survive any operation cost shock realization can survive any marginal decrease in profit or subsidy, and its mass does not change. In this way, (3.8) maps the change in profit to the change in mass of the firm.

3.3.2 Example with an Economy with Two Types of Firms

To better understand how the propagation of shocks depends on the network structure, we now show three specific examples of network disruptions and their propagation due to policy change shocks under three particular network structures. For simplicity, we assume only two types of firms exist: small and large firms with $\theta_1 \neq 0$, $\theta_2 = 0$ and $\boldsymbol{M} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

[22] Here we abuse the $+$ symbol as $\boldsymbol{x}+1 \equiv (x_1+1, \cdots, x_n+1)$ and the fraction bar $-$ as $\boldsymbol{x}/\boldsymbol{y} \equiv (x_1/y_1, \cdots, x_n/y_n)$ or $\equiv \boldsymbol{x} \circ \boldsymbol{y}^{\circ -1}$ in the hadamard expression.

[23] Strictly speaking, the pdf with upper-bounded support has an indifferentiable point. We assume π_i is not at such a point and θ_i is well-defined because the probability that π_i takes a specific value is 0 given finite n.



(a) Positive assortative economy (b) Small customer economy (c) Small supplier economy

Figure 6: Economy with two types of firms

Each vertex corresponds to a firm type (filled with a continuum of firms with the type). A directed edge from one vertex to another implies the former supplies IMD goods to the latter. In the positive assortative economy (a), there are networks only between small firms and ones between large firms. In the small customer economy (b), small firms function as customers but not as suppliers. In the small supplier economy (c), small firms function as suppliers but not as customers.

3.3.2.1 Positive Assortative Economy Firstly, to clarify direct/indirect network effects and their upstream/downstream propagation, we analyze a positive assortative economy in the sense that small firms connect to small firms and large firms connect to large firms as shown in Figure 6a.

For simplicity, we assume $\boldsymbol{m} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. From the definition, network matrix is $\boldsymbol{N} = \boldsymbol{N_s} = \boldsymbol{N_c} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. In this positive assortative economy, there are networks only between small firms and ones between large firms. Since

$$I + \beta \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \beta^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^2 + \cdots = \underbrace{I}_{\text{Direct Network Effect}} + \underbrace{\begin{pmatrix} \frac{\beta}{1-\beta} & 0 \\ 0 & \frac{\beta}{1-\beta} \end{pmatrix}}_{\text{Indirect Network Effect}} \tag{3.9}$$

holds, and from $\theta_2 = 0$, upstream/downstream propagation can be written as follows.

$$\begin{pmatrix} d\Phi_1 \\ d\Phi_2 \end{pmatrix} = \begin{pmatrix} 1 + \frac{\beta_\Phi}{1-\beta_\Phi} & 0 \\ 0 & 1 + \frac{\beta_\Phi}{1-\beta_\Phi} \end{pmatrix} \beta_\Phi \begin{pmatrix} \Phi_1 dM_1 \\ 0 \end{pmatrix} \tag{3.10}$$

$$= \begin{pmatrix} \left(\underbrace{1}_{\text{Direct Network Effect}} + \underbrace{\frac{\beta_\Phi}{1-\beta_\Phi}}_{\text{Indirect Network Effect}} \right) \beta_\Phi \Phi_1 dM_1 \\ 0 \end{pmatrix} \tag{3.11}$$

$$\begin{pmatrix} d\Delta_1 \\ d\Delta_2 \end{pmatrix} = \begin{pmatrix} \left(1 + \frac{\beta_\Delta}{1-\beta_\Delta}\right) \beta_\Delta \Delta_1 dM_1 \\ 0 \end{pmatrix} \tag{3.12}$$

Note that since there is no direct/indirect connection between the exiting 1-firms, productivity and scaled demand of 2-firms do not change.

The change in Φ_1 shown in (3.11) and the change in Δ_1 shown in (3.12) can be graphically understood well as follows. Figure 7 and Figure 8 express what happens when the supplier/customer is on the production network for there is no heterogeneity like Figure 2 and Figure 3.

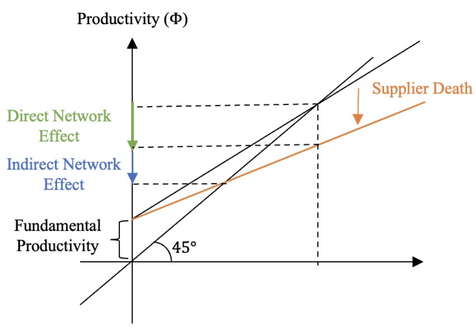


Figure 7: Supplier death effect

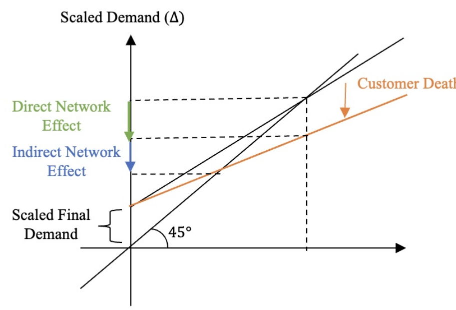


Figure 8: Customer death effect

Let's focus on the supplier death effect shown in Figure 7. Once the subsidy is decreased and a fraction of 1-firms is forced to exit, surviving 1-firms have to produce with fewer suppliers, directly decreasing their productivity. This is the direct network effect shown as the green line and expressed in the first term of (3.4) and (3.11). However, once its productivity declines, its customers, in turn, suffer from the higher cost of an intermediate good and also decrease productivity, which continues to propagate with attenuation. This is an indirect network effect shown as the blue line, expressed in the second term in (3.4) and (3.11). In this way, the supplier death effect propagates downstream via the production network. Also for the customer death effect, there is upstream propagation via the production network.

Next, let's focus on the change in profit and mass of operating firms. From (3.7) and (3.8), change in profit and mass of small firms can be expressed as

$$\frac{d\pi_1}{\pi_1} = \frac{d\Delta^H}{\Delta^H} + \frac{d\Delta_1}{\Delta_1} + \frac{d\Phi_1}{\Phi_1} \tag{3.13}$$

$$dM_1 = \theta_1^* \left(\frac{d\pi_1}{\pi_1} + \frac{ds}{\pi_1} \right) \tag{3.14}$$

where $\theta^* = \theta \dfrac{\pi^*}{\pi}$, and this implies

$$\frac{d\pi_1}{\pi_1} = \frac{d\Delta^H}{\Delta^H} + \underbrace{\left(\left(1 + \frac{\beta_\Phi}{1-\beta_\Phi} \right) \beta_\Phi + \left(1 + \frac{\beta_\Delta}{1-\beta_\Delta} \right) \beta_\Delta \right) dM_1}_{\text{Network Effect}} \tag{3.15}$$

$$dM_1 = \theta_1^* \left(\frac{d\Delta^H}{\Delta^H} + \frac{ds}{\pi_1} \right) + \theta_1^* \underbrace{\left(\left(1 + \frac{\beta_\Phi}{1-\beta_\Phi} \right) \beta_\Phi + \left(1 + \frac{\beta_\Delta}{1-\beta_\Delta} \right) \beta_\Delta \right) dM_1}_{\text{Network Effect}}. \tag{3.16}$$

(3.16) highlights the effects of network destruction on the mass of small firms. Without it, change in the mass is expressed only as the first term in (3.16), which simply captures GE effects and change in subsidy. In this networked economy, however, firms' exit destructs the network and accelerates exits, as expressed in the second term. This suggests that subsidy policy reforms aiming at reducing the mass of small firms may result in *excess exits* in the short run due to the destruction of the production network.

3.3.2.2 Negative Assortative Economy Secondly, to clarify how the exit of small firms affects large firms, suppose a negative assortative economy in the sense that small firms tend to connect to large firms. We will relate the two economies below to the calibrated production network structures in Section 5.

3.3.2.2.1 Small Customer Economy First, we analyze an economy with $\boldsymbol{m} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$, which results in $\boldsymbol{N_s} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ and $\boldsymbol{N_c} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$. In this economy, small firms function as customers but not as suppliers in the production networks, as shown in Figure 6b. Then, upstream/downstream propagation can be written as follows.

$$\begin{pmatrix} d\Phi_1 \\ d\Phi_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3.17}$$

$$\begin{pmatrix} d\Delta_1 \\ d\Delta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \left(\underbrace{1}_{\text{Indirect network Effect}} + \underbrace{\frac{\beta_\Delta}{1-\beta_\Delta}}_{\text{Direct network Effect}} \right) \beta_\Delta \Delta_1 dM_1 \end{pmatrix} \tag{3.18}$$

In this economy, since small firms do not supply intermediate goods, the upstream network effect via the production network does not appear. On the other hand, customer death shock propagates upstream and hits large firms. Furthermore, since large firms are customers of large firms themselves, the indirect network effects propagate among large firms.

3.3.2.2.2 Small Supplier Economy Next, we analyze a economy with $\boldsymbol{m} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$, which results in $\boldsymbol{N_s} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$ and $\boldsymbol{N_c} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$. In this economy, small firms function as suppliers, but not as customers, as shown in Figure 6c. Then, upstream/downstream propagation can be written as follows.

$$\begin{pmatrix} d\Phi_1 \\ d\Phi_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \left(1 + \frac{\beta_\Phi}{1-\beta_\Phi} \right) \beta_\Phi \Phi_1 dM_1 \end{pmatrix} \tag{3.19}$$

$$\begin{pmatrix} d\Delta_1 \\ d\Delta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3.20}$$

Note that in this economy, since small firms do not purchase intermediate goods, the downstream network effect does not appear. On the other hand, supplier death shock propagates downstream and hits large firms, propagating among large firms.

3.4 Propagation Effects on Welfare

Lastly, we see the effect of policy change on welfare in the networked economy. We show that in addition to the usual reallocation effect, two additional terms arise which reflect downstream/upstream propagation, respectively.

Proposition 3 (Welfare change in networked economy)**.** Given the changes in the amount of operation labor, the mass of firms, scaled demand, and productivity, the change in welfare can be decomposed as

$$\frac{dU}{U} = -\underbrace{\left(w^o \frac{dL^o}{L^o} + \boldsymbol{w^{L}}' \frac{d\boldsymbol{M}}{\boldsymbol{M}}\right)}_{\text{Reallocation from exiting firms}} - \underbrace{\boldsymbol{w^{L}}' \frac{d\boldsymbol{\Delta}}{\boldsymbol{\Delta}}}_{\text{Reallocation from IMD suppliers}} + \underbrace{\frac{\sigma}{\sigma-1}\boldsymbol{w^{E}}' \left(\frac{d\boldsymbol{M}}{\boldsymbol{M}} + \frac{d\boldsymbol{\Phi}}{\boldsymbol{\Phi}}\right)}_{\text{Change in CPI}} \tag{3.21}$$

where $w^o = \dfrac{L^o}{L^p}$ is the ratio of operation labor to production labor in the entire economy, $w_i^L = \dfrac{l_i M_i}{L^p}$ is the production labor share of i-firms, and $w_i^E = \dfrac{p_i x_i^H M_i}{P^H U}$ is household's expenditure share on i-firms' goods.

Proposition 3 highlights the welfare implication of network destruction and its propagation. To relate the results to Proposition 1 and 2, firstly, suppose there is no production network ($\boldsymbol{N} = \boldsymbol{0}$) . From Proposition 1 and 2, this implies neither productivity nor scaled demand changes ($d\boldsymbol{\Phi} = d\boldsymbol{\Delta} = \boldsymbol{0}$) because a firm's exit does not have any externality on other firms except for aggregate effects. In this situation, equation 3.21 captures a standard trade-off between labor reallocation and loss of variety in misallocation literature like Dhingra and Morrow (2019). Given the decline in the mass of firms, the first two terms capture the reallocation of labor resources from exiting firms to surviving firms. This has a positive effect on welfare. The fourth term captures the loss of variety due to exiting firms and the resulting increase in CPI. This has a negative effect on welfare.

Under the networked economy with $\boldsymbol{N} \neq \boldsymbol{0}$, however, two additional effects work on welfare due to downstream/upstream propagation of network destruction. First, suppose exiting firms were functioning as suppliers. In this situation, Proposition 1 implies that downstream propagation decreases the productivity of surviving firms ($d\boldsymbol{\Phi} < \boldsymbol{0}$) and increases CPI alongside the loss of variety effects. Thus, the fifth term in equation 3.21 decreases welfare. Second, suppose exiting firms were functioning as customers. In this situation, Proposition 2 implies that upstream propagation decreases scaled demand of surviving firms ($d\boldsymbol{\Delta} < \boldsymbol{0}$) . As shown in Figure 8, the decline in scaled demand is caused by the decline in scaled IMD demand. From equation (2.26), this decreases labor demand to produce IMD goods, which are not consumed by the household, and the released labor is reallocated to produce final goods, which are consumed by the household.[24] Thus, the third term in equation 3.21 increases welfare.

[24] If we set CPI as numeraire instead of wage, the decline in scaled IMD demand is observed as a decline in wage.

4 Endogenous Network

The analysis so far clarified how the destruction of the network by the exit of small firms propagates to the entire economy given an exogenous network structure. It revealed short-run policy effects at the level where the network structure is fixed. In reality, however, it is reasonable to expect the network is reconstructed in response to policy changes. In an economy that is harsh on small firms, the network structure will bypass small firms. In this section, we endogenize network formation by extending the idea of the costly-relationship model adopted by Lim (2018) and Bernard et al. (2022).

4.1 Costly-relationship

We assume the construction of a customer-supplier relationship between two firms requires *relationship cost* which is a random variable $\xi \underset{i.i.d.}{\sim} G_\xi(\xi)$ in units of *network labor*. In the economy, L^f units of network labor are inelastically supplied by the household, and its wage (*network wage*) is determined so that the network labor market clears.[25]

One crucial assumption here is that the cost for link creation is paid by a supplier, like an exporter decision of Melitz (2003). Under this assumption, there is no incentive for a potential customer to decline an offer of link-creation by a supplier because of the love of variety structure of the customer's production function.

After the relationship cost is realized, the supplier makes an activation decision comparing the benefit via the network and the realized relationship cost. We assume that a χ'-supplier does not consider changes in other firms' behavior when it creates a new relationship between a χ-customer.[26]

4.1.1 Profit from Each Relationship

The benefit a supplier can obtain from a relationship is the same as the profit from selling intermediate goods to the customer. This is because increasing customers does not change the supplier's productivity[27] due to the constant return to scale production function. If χ'-supplier make a customer-supplier relationship between a χ-customer, from (2.8) and (2.9), the χ'-supplier obtains the profit as follow.

$$\pi(\chi, \chi') = \mu^{-\sigma}(\mu - 1)\Delta^H \Delta(\chi)\Phi(\chi') \tag{4.1}$$

[25] These multiple types of labor is observed in several kinds of literature. For example, Acemoglu et al. (2018) assumes the representative household inelastically supplies some unit of labor which is used in the production and some unit of labor which is used for operation fixed cost and R&D in the model.

[26] Ignoring strategic interactions in network formation are widely adopted in the literature of network formation under GE framework like Lim (2018), and Bernard et al. (2022). Acemoglu and Azar (2020) also incorporates a similar condition as *contestabile market structure*, which excludes such an interplay in its equilibrium definition. Oberfield (2018), however, suggests a new GE concept like *N-stable equilibrium* where there are no coalitions of N firms with dominating deviations over networks, which accommodates the strategic interplay of firms. We drop out of such a strategic interplay in this model to simplify the following discussion.

[27] This comes from the competitive assumption mentioned above. Once a χ-supplier obtains additional customers, it becomes a more attractive customer for other potential suppliers, which results in larger suppliers and higher productivity of the supplier in equilibrium. If we do not impose the competitive assumption, the χ-firm must consider the productivity gain as well as a simple increase in its customer, which is hard to analyze.

4.1.2 Network Formation

Given a CDF of relationship costs $G_\xi(\xi)$, a matching function that returns the probability that a potential relationship between a χ'-supplier and a χ-customer is activated can be written as

$$m(\chi, \chi') = Pr\left[\pi(\chi, \chi') > w^f \xi\right] \tag{4.2}$$
$$= G_\xi\left(\frac{\pi(\chi, \chi')}{w^f}\right). \tag{4.3}$$

Given the decision-making about network formation above, relationship costs actually paid by χ'-supplier is

$$c^f(\chi') = w^f l^f(\chi') \tag{4.4}$$

where $l^f(\chi')$ denotes network labor hired by χ'-firm and given by

$$l^f(\chi') = \int_\chi \int_{-\infty}^{\pi(\chi,\chi')} \xi dG_\xi(\xi) dM(\chi). \tag{4.5}$$

Network wage is determined so that the network labor market below clears.

$$L^f = \int l^f(\chi') dM(\chi') \tag{4.6}$$

4.1.3 Example of an Endogenous Matching Function

Figure 9 shows an example of the realization of productivity, scaled demand, the mass of supplies, and the mass of customers in a steady state through numerical simulation. Just for simplicity, we assume the covariance between ϕ and δ is zero. Firstly, we can confirm a strong positive correlation between productivity and ϕ. This is natural because the productivity of a firm is partly determined by its fundamental productivity, as shown in Figure 2. Then, since a firm with high productivity can sell a large amount of IMD goods to its customers, this increases the value of a network creation of the firms as shown in (4.1). Thus, it creates many links and obtains many customers, as shown in the bottom-left panel. Secondly, we can confirm a strong positive correlation between scaled demand and δ. This is natural because the scaled demand of a firm is partly determined by its fundamental demand, as shown in Figure 3. Then, since a firm with large scaled demand can purchase a large amount of intermediate inputs, it attracts many suppliers, as shown in the bottom-right panel. Thirdly, we can confirm weak positive correlations between productivity and δ, and between scaled demand and ϕ. A firm with large fundamental productivity obtains a lot of customers, increasing its scaled demand, and a firm with large fundamental demand obtains a lot of suppliers, increasing its productivity.

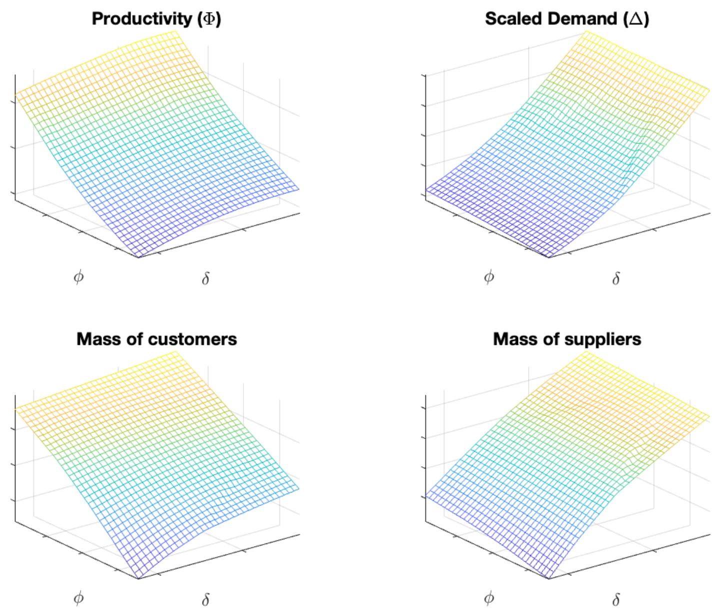


Figure 9: Network characteristics and fundamental heterogeneity

4.2 Equilibrium

Lastly, we define an equilibrium concept that endogenizes network formation.

Definition 2 (Endogenous network equilibrium)**.** Given a subsidy policy s, an endogenous network equilibrium is allocation functions $\Delta(\chi), \Delta^H$, price function $\Phi(\chi)$, a distribution function $M(\chi)$, matching function $m(\chi, \chi')$ and network wage w^f, such that (i) consumer chooses consumption to maximize utility, (ii) potential firms construct production network and make stay/exit decision to maximize its value (iii) firms choose production input, price to maximize its profit, (iv) all markets clear.

Showing the uniqueness of this endogenous network equilibrium is not as simple as in the case of exogenous network equilibrium because Blackwell's sufficient conditions are not satisfied. Even so, as long as the assumption on β_Φ and M_e discussed in Section 2 holds, backward and forward fixed point equations (2.21) and (2.22) have unique solutions for any structure of the endogenously determined networks. In fact, numerical simulations reveal no counterexample even starting from different initial guesses of equilibrium variables.

5 Computational Exercise

In this section, we first parameterize the model developed in section 2 and section 4. We next analyze the dynamic impact of changes in subsidy policy on the economy. We assume that the change in subsidy policy is not anticipated and that the timing of the reform is the same as in section 5: networking has been completed, affecting only firms' exit and production decisions.

To make the discussion clear, we define three states of the economy: *subsidy state* at $t = 0$, where subsidy covers operation costs, *transition state* at $t = 1$, where subsidy is unexpectedly stopped after networking, *reformed state* at $t \geq 2$ where production networks are re-constructed given the subsidy policy has already stopped. All the states are respectively well-defined by the equilibrium concepts defined in Section 2 and Section 4. The subsidy state at $t = 0$ is endogenous network equilibrium with subsidy $s = s_0 > 0$. The transition state at $t = 1$ is exogenous network equilibrium with reduced subsidy ($s = s_1 < s_0$), and matching function m and network formation cost c^f are succeeded from subsidy state. The reformed state at $t \geq 2$ is endogenous network equilibrium with reduced subsidy ($s = s_1 < s_0$). Note that there is no stickiness in this economy, except for the production network, which cannot be updated due to timing discrepancies of the reform. Hence, this economy arrives at its after-reform steady state at $t = 2$. Hereafter, we call $t = 1$ as the short run and $t \geq 2$ as the long run interchangeably.

To clarify a relationship between network structures and how the shock of the policy change propagates, we consider two distinct economies: *ϕ-economy* where firms are heterogeneous in its fundamental productivity ϕ and fundamental demand δ is common across firms, and *δ-economy* where firms are heterogeneous in its fundamental demand δ and productivity ϕ is common across firms. Note that without a production network, both economies appropriately parameterized return totally the same response to the policy change. However, in a networked economy, due to the difference in the position on the production network, the two economy returns totally different responses to the policy shock. In the following parts of this section, we conduct calibration and simulation on both economies and compare the results to emphasize the role of small firms in each network structure.

5.1 Parameterization

To perform policy simulations, we first parameterize the model so that aggregate variables in the subsidy state roughly fit the actual aggregate value. First, we discuss the demand parameters. We set the elasticity of substitution σ as 3 like Hsieh and Klenow (2009), which is commonly used in the literature.[28] We normalize total general labor supply $L = 0.93$ and network labor supply $L^f = 0.07$ so that 7% of all the labor ($L + L^f = 1$) in the economy is devoted to network formation as estimated in an earlier version of Lim (2018).

Secondly, we discuss the production parameters. M_e is set as 2 just for simplicity. This satisfies the assumption for uniqueness discussed in Section 2 combined with the value of elasticity of substitution above. We assume operation cost follows a normal distribution with variance 0.05[29] and mean of it as 0.15. This results in 35% of all the labor being used for production in both economies, following the share of professional and engineering workers and clerical workers in the workforce being 35% in Japan.[30]We assume firm fundamental productivity and fundamental demand follow log-normal distribution independently. In ϕ-economy, we set $\log(\phi) \underset{i.i.d.}{\sim} \mathcal{N}(0, 1)$ and $\log(\delta) = 0$, and in δ-economy, we set $\log(\delta) \underset{i.i.d.}{\sim} \mathcal{N}(0, 0.85)$ and $\log(\phi) = 0$ for simplicity. This assumption on

[28]Lim (2018) estimate the parameter and report 2.7 in ϕ-economy under the same assumption of our model that firms and households have the same elasticity of substitution.

[29]While this pdf does not have upper bounded support, due to the small variance, θ defined in Section 5 is almost 0 for large firms.

[30]See `https://www.stat.go.jp/english/data/roudou/report/2015/index.html`.

variance implies the firm with average sales in the economy is located in the 80 percentile of the firm size distribution in both economies following distribution of SMEs where firms with average sales are located in from the 75 percentile to the 90 percentile.[31]

Thirdly, we discuss the network parameters. We assume network formation cost follows a uniform distribution with $\xi \underset{i.i.d.}{\sim} U[0, 0.075]$.[32] This results in the final good share being 66% in ϕ-economy and 65% in δ-economy following actual final goods share 64% in Japan.[33]

Lastly, we discuss the subsidy policy s. We set $(s_0, s_1) = (0.6, 0.45)$ in ϕ-economy, and $(s_0, s_1) = (0.9, 0.75)$ in δ-economy. At subsidy state $(t = 0)$ in both economies, the setting results in a mean of the survival rate h of 0.95 following the annual business exit rate of 95% in Japan. At reformed state $(t = 2)$, the setting results in a mean of the survival rate h of 0.90 following the annual business exit rate of 90% in the U.S.[34]

5.2 Steady States

Before observing dynamics, we first check the structure of the network in both economies to grasp how small firms' exit shock propagates to the economy via the network. Figure 10 and 11 show the customer network matrix $\boldsymbol{N_c}$, supplier network matrix $\boldsymbol{N_s}$, downstream influence matrix $\boldsymbol{R_\Phi}$, and upstream influence matrix $\boldsymbol{R_\Delta}$ sorted by firm size in ϕ-economy and δ-economy, respectively. As defined in Section 5, (x, y)-element of $\boldsymbol{N_s}$ corresponds to x-firm's mass of y-suppliers and (x, y)-element of $\boldsymbol{N_c}$ corresponds to x-firm's mass of y-customers. (x, y)-element of $\boldsymbol{R_\Phi}$ corresponds to x-firm's mass of direct and indirect y-suppliers, which determines the entire downstream propagation effect of y-suppliers' exit. (x, y)-element of $\boldsymbol{R_\Delta}$ corresponds to x-firm's mass of direct and indirect y-customers, which determines the entire upstream propagation effect of y-customers' exit.

First, both in Figure 10 and Figure 11, we see that the rightward and downstream direction is brighter (larger values) in the supplier/customer network matrices, which indicates that larger firms have more customers and suppliers, and especially more networks with smaller firms. This mainly reflects the situation that only large firms can connect with small firms, a feature confirmed by many observational studies using production network data.[35] This implies the direct network effect of small firms' exit mainly hits large firms.

Next, let's focus on the difference in the network structures in both economies to evaluate which direction upstream or downstream network effects play an essential role as a propagation mechanism after a policy reform in the economies. First, we focus on ϕ-economy (Figure 10). Since small firms have the same fundamental demand δ as large firms, small firms are attractive customers, which can be checked as the upper right is bright in the $\boldsymbol{N_s}$ matrix. This implies suppliers of small firms

[31] See `https://www.chusho.meti.go.jp/pamflet/hakusyo/2021/PDF/chusho/03Hakusyo_part1_chap2_web.pdf`. (In Japanese)

[32] Lim (2018) uses log-normal distribution and Huneeus (2020) uses Weibull distribution, both of which have two parameters to structurally estimate endogenous network model similar to the model using micro firm panel data. Since we don't use microdata and calibrate the model based on aggregate variables, we adopt the uniform distribution, which has only one parameter to reduce the number of estimand parameters.

[33] See `https://www.soumu.go.jp/english/dgpp_ss/data/io/io15_00001.htm`.

[34] See `https://www.chusho.meti.go.jp/pamflet/hakusyo/2021/PDF/chusho/03Hakusyo_part1_chap2_web.pdf`. (In Japanese)

[35] Negative assortative matching is confirmed by Lim (2018) for the listed companies' data in the U.S., by Bernard et al. (2019) for Japanese buyer-seller relationships data, by Bernard et al. (2022) for Belgian VAT data, and by Antras and Chor (2021) for a global production network. We also confirmed the negative assortativity quantitatively with respect to degree and scale in both economies. (Upstream firm size (revenue) assortativities are -0.58 and -0.46 respectively in ϕ-economy and δ-economy, and downstream assortativities are -0.79 and -0.46, respectively.)

are large firms. However, since small firms have lower ϕ than large firms, they are unprofitable suppliers and cannot supply intermediate goods to large firms. This can be checked as the upper rows are totally dark in the $\boldsymbol{N_c}$ matrix. These characteristics of the network matrices are succeeded to the influence matrices $\boldsymbol{R}$, which are series of each network matrix. Hence, in this economy, the effects of small firms' exit mainly propagate upstream to large suppliers as customer death, as shown in the brightness of the lower left part in $\boldsymbol{R_\Delta}$. You can confirm the effect does not propagate downstream by checking left columns are totally dark in $\boldsymbol{R_\Phi}$. Remember that this corresponds to the small customer economy in the example 3.3.2.2 with $\boldsymbol{N_s} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ and $\boldsymbol{N_c} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$. While we exogenously gave the network structure and derived the propagation there, the corresponding microfoundations for the network structure are given here.

In the δ-economy (Figure 11), on the other hand, all the matrices look like the transposed matrices in Figure 10. Since small firms function as suppliers, not as customers in this δ-economy, the effect of exit of small firms propagates downstream as supplier death, as shown in the brightness of the lower left part in $\boldsymbol{R_\Phi}$. You can confirm the effect does not propagate upstream by checking left columns are totally dark in $\boldsymbol{R_\Delta}$. This corresponds to the small supplier economy in the example 3.3.2.2.2 with $\boldsymbol{N_s} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$ and $\boldsymbol{N_c} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$.

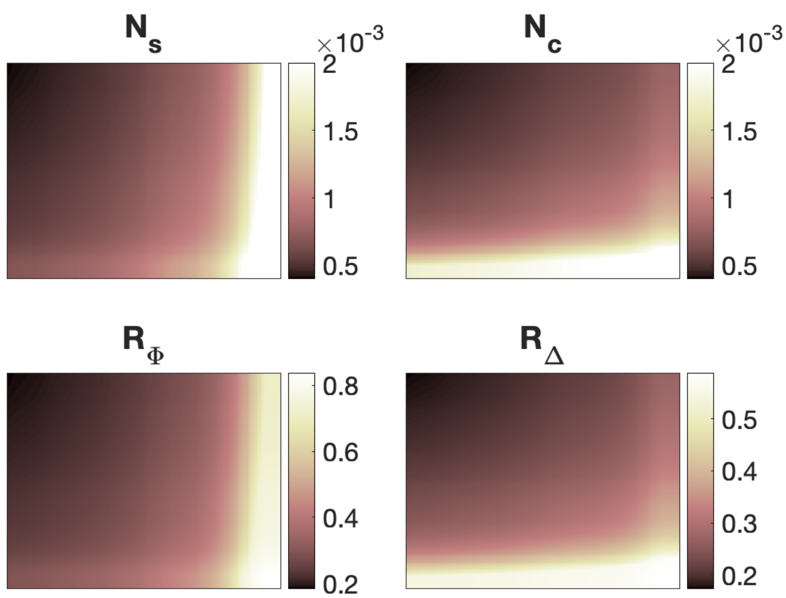


Figure 10: Network and influence matrix in ϕ-economy

$\boldsymbol{N_s}(x,y)$ and $\boldsymbol{N_c}(x,y)$ correspond to x-firm's mass of y-supplier/customer. $\boldsymbol{R_\Phi}(x,y)$ and $\boldsymbol{R_\Delta}(x,y)$ corresponds to x-firm's mass of direct and indirect y-suppliers/customers, which determines the entire downstream/upstream propagation effect of y-suppliers/customers' exit on x-firm. this corresponds to the small customer economy in the example 3.3.2.2 with $\boldsymbol{N_s} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ and $\boldsymbol{N_c} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$.

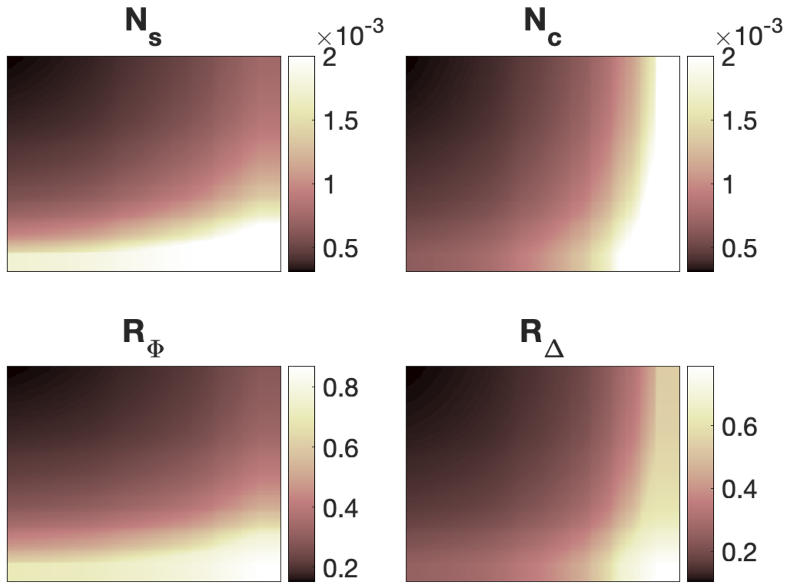


Figure 11: Network and influence matrix in δ-economy

$\boldsymbol{N_s}(x,y)$ and $\boldsymbol{N_c}(x,y)$ correspond to x-firm's mass of y-supplier/customer. $\boldsymbol{R_\Phi}(x,y)$ and $\boldsymbol{R_\Delta}(x,y)$ corresponds to x-firm's mass of direct and indirect y-suppliers/customers, which determines the entire downstream/upstream propagation effect of y-suppliers/customers' exit on x-firm. This corresponds to the small supplier economy in the example 3.3.2.2.2 with $\boldsymbol{N_s} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$ and $\boldsymbol{N_c} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$.

5.3 Dynamics

Having calibrated the parameters of the model and checked the structure of the network in the subsidy state, we now assess the key question of this paper: What are the dynamic implications of the reform in preferential policies for SMEs in a networked economy? To answer the question, we first show the dynamics of the aggregate variables of the model in response to the counterfactual policy changes and next decompose these changes following Proposition 3 to clarify the way of propagation. We do this for the two economies and check how the different assumptions on the source of heterogeneity and associated network structures alter the welfare implication of the policy reform.

5.3.1 ϕ-economy

Figure 12 shows the dynamic responses of aggregate variables following the subsidy policy reform in ϕ-economy. The blue lines correspond to responses when the policy change is unanticipated as defined so far. We also plot the responses when the policy change is anticipated before networking, and the network is constructed to accommodate the policy change for reference. All values are expressed as a rate of change from subsidy state ($t = 0$). $t = 1$ corresponds to transition state and $t \geq 2$ corresponds to reformed equlibrium.

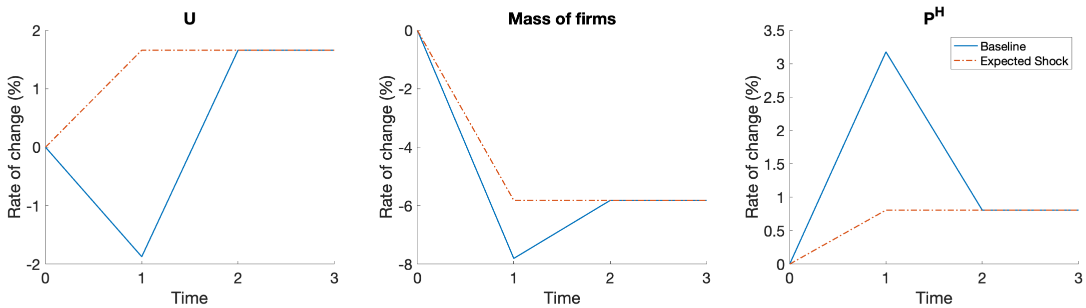


Figure 12: Dynamics of aggregate variables in ϕ-economy

Firstly, in welfare, we see a strong J-curve effect of a short-run decrease and a long-run increase, as shown in the blue lines. This means that the policy change is welfare-improving in the long run, but the side effects of network destruction are strong enough to surpass the reallocation effects in the short run. This is an important finding because it means policymakers who try to implement measures that remove the wedge causing misallocation will have to tolerate a short-run recession. Secondly, in the mass of firms, after a serious short-run decrease, it recovers to a position lower than the original level in the long run. This means that the excess exit effect due to chain bankruptcies through the networks discussed in (3.16) is at work. The final mass of firms declines by about 6%, but in the short run, they decline by about 2ppt more. The long-run decline is not surprising, which is a consequence of reduced subsidies. Thirdly, in CPI, after a significant short-run increase, it declines to a position higher than the original level in the long run. Remember that the change in CPI is determined by the change in the mass of firms due to the love of variety effect and the change in the productivity of each firm. Since the reform reduces the mass of firms by expelling small firms, the loss of variety effect increases CPI at $t \geq 1$. In the short run, productivity declines due to the destruction of networks and its downstream propagation as supplier death. After production networks are reconstructed at $t \geq 2$, CPI becomes lower than $t = 1$ in the long run. Finally, we point out the importance of announcing policy changes. Because there are no sticky variables in this abstract economy, if policy changes are announced before network construction, the economy can immediately transition to a new, better steady state and avoid short-run welfare losses as shown in the red dashed lines. Of course, in reality, many stickiness, such as labor market inflexibility and capital adjustment costs, will prevent such an immediate adjustment. Still, this result suggests the quantitative significance of the adjustments of production networks on welfare after policy reforms.

To clarify the differences in the propagation paths of the policy change due to the different production network structures assumed in ϕ-economy and δ-economy, we next decompose the dynamic welfare change based on the formula defined in Proposition 3.[36] The stacked bar chart in Figure 13 shows the contribution of various components to the change in welfare (solid black lines).

[36] While this decomposition is valid only when the changes are so small that we can disregard second-order difference, it seems to decompose the changes well.

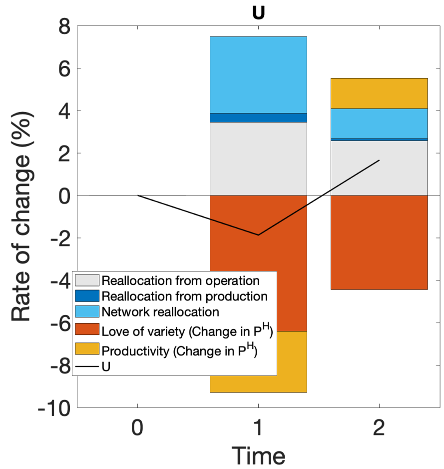


Figure 13: Decomposition of change in welfare in ϕ-economy

In the short-run, two conventional reallocation effects, reallocation from operation and production of exiting firms, are positive and contribute to welfare improvement. This result is consistent with the basic literature of misallocation, which points out that reallocation from excess small firms to larger profitable firms improves welfare. Note that the effect of reallocation from production is small. This is because exiting small firms in ϕ-economy have low labor productivity, and do not use labor input so much as intermediate inputs. Hence, they do not release labor resources so much after the reform. Instead, the network reallocation effect is stark. As Figure 10 shows, large firms supply intermediate goods to small firms in the economy. As those customers exit after the reform, large firms can save the labor they used to make the intermediate goods for exiting small firms. Since the labor resources are reallocated for final goods production, it improves welfare. All the results are consistent with the observation of influence matrices in 10 and also consistent with the observation in Example 3.3.2.2 that large (surviving) firms are hit by the upstream network effect, and their scaled demand declines. The short-run increase in CPI is mainly due to the loss of variety. Since even small firms have the same fundamental demand δ as large firms in this economy, small firms' final good composes a large share of the household's expenditure. The sudden exit of small firms and the loss of the variety of final goods damage the price index of the household seriously. This logic can be confirmed by the third bracket in (3.21), which expresses the damage to the welfare of loss of variety effects as the inner product of expenditure share and change in mass of firms.

The difference between short-run and long-run welfare is driven mainly by two factors. The first one is the reconstruction of the production networks. Considering the policy reform which decreases the survival rate of small firms, potential suppliers try to avoid relationships between them as expressed in (4.1). This results in a synergistic increase in the productivity of large firms thanks to the spillover via the production networks. We can confirm this effect by the positive contribution of productivity to welfare at $t = 2$. The second one is the resolve of excess exits. As observed in Figure 12, the mass of firms declines too much at $t = 1$ in comparison with $t = 2$. This decreases reallocation from operation and production but increases the love of variety effect.

5.3.2 δ-economy

Figure 14 shows the dynamic responses of aggregate variables in δ-economy. Qualitatively, the dynamics are similar to ϕ-economy. Welfare follows the J-curve, firms exit excessively in the short run, and CPI rises sharply in the short run and declines to a level higher than its original level.

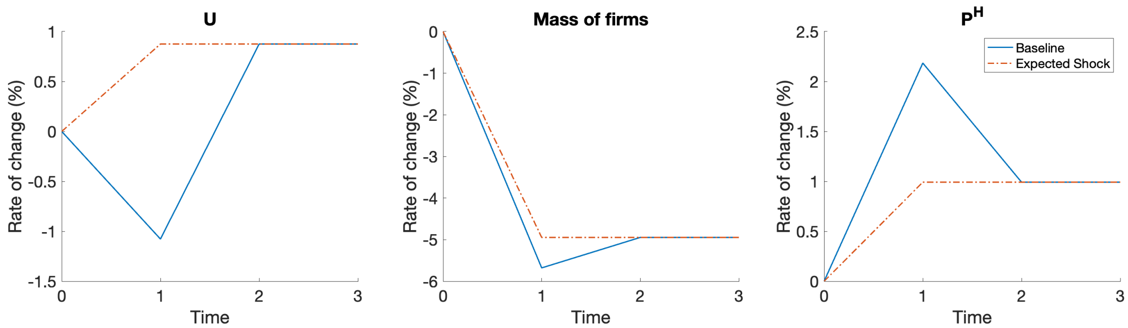


Figure 14: Dynamics of aggregate variables in δ-economy

Figure 15 shows the decomposition of the change in welfare in δ-economy. In the short run, we can confirm that the decline in productivity due to downstream propagation raises CPI and significantly reduces welfare in the short run. This is because small firms in this δ-economy function as suppliers, as checked in the structure of the network matrix in Figure 11. The sudden policy reform causes suppliers' death, and it propagates upstream via the remaining production networks. This is consistent with the observation of influence matrices in Figure 11 and Example 3.3.2.2.2 that large (surviving) firms are hit by the downstream network effect. Since small firms do not function as customers in this economy, the effect of upstream propagation is very weak. Instead, reallocation from production exerts upward pressure on welfare. Since small firms have the same fundamental productivity as large firms, the labor inputs of small firms in this δ-economy are not so small as in ϕ-economy. The exit of these small firms releases the production labor a lot.

In the long run, network reconstruction recovers productivity, and welfare is improved more than the subsidy state. It is worth mentioning that productivity does not recover up to the original level even after network reconstruction. This is in contrast to ϕ-economy, where productivity has risen in the long run above original levels. To understand the mechanism of the results, we need to mention two different impacts of the exit of small firms on the new supplier networks of large firms. The first is an increase in the average supplier productivity. Since small firms function as suppliers in this economy, their exit reduces the demand for network labor, which lowers network wages. This allows large suppliers to use more network labor, creating a network with large customers that complements the loss of original small suppliers for the customers. Though this increases the average size and productivity of the suppliers, the customer's productivity gain is small because small firms and large firms have the same fundamental productivity in this δ-economy. The second is the inefficient use of network labor. As the network wage decreases, the reservation relationship cost increases. This implies an increase in average relationship cost per unit of activated networks. The inefficient use results in a decrease in the mass of networks in the economy, and productivity gain from the spillover of suppliers' productivity is decreased in the entire economy. In the equilibrium, the second negative effect surpasses the first positive effect, and the productivity is decreased even in the long run, where the production network is reconstructed.

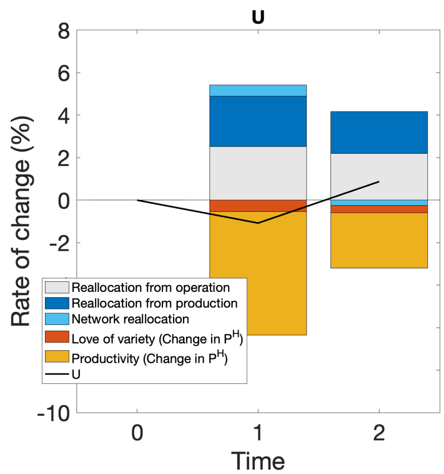


Figure 15: Decomposition of change in welfare in δ-economy

6 Conclusion

This paper provides a new model of production network construction and destruction in a portable way and offers a simple analytical expression about the effect of network destruction on the entire economy and how it propagates. The computational exercise reveals that while the subsidy policy reform can improve welfare in the long run, it is accompanied by a serious short-run decline in welfare due to the destruction of the production network. We also confirm that the propagation paths of the reform vary greatly depending on the production network structure.

The analysis in this paper provides different scope for future research. Firstly, we expect that estimating this model using real inter-firm network data, which has been increasingly used in the literature in recent years, will provide more realistic suggestions. In particular, the network matrix and influence matrix introduced in this paper can be directly observed by firm size and its network information without any distributional assumption on firms' productivity. As confirmed in Section 5, the matrix helps to forecast how the effects of network destruction due to small firms' exit propagate to the entire economy. Hence, the estimated matrix would be a valuable tool for policymakers who actually implement size-dependent policies.

Secondly, the analytical exercise of this research is mainly on the propagation of shocks, and the analytical predictions on network reconstruction are not enough. Given the different welfare implications resulting from different assumptions on the source of heterogeneity and associated network structures revealed in Section 5, the analytical explanation will lead to a fruitful discussion about reallocation policy in a networked economy. In fact, we believe that there is plenty of room in the model for detailed steady-state analysis with endogenous network reconstruction, and it is left for future research.

Lastly, macroeconomic research on inter-firm networks is still in its infancy compared to the fields of input-output analysis and social network analysis. Starting from the analytical results of this study, which point out similarities with the concepts in those fields, we expect to gain various perspectives by actively incorporating previous research in these fields.

References

Acemoglu, Daron, Ufuk Akcigit, Harun Alp, Nicholas Bloom, and William Kerr. 2018. "Innovation, reallocation, and growth." *American Economic Review* 108 (11): 3450–91.

Acemoglu, Daron, Ufuk Akcigit, and William Kerr. 2016. "Networks and the macroeconomy: An empirical exploration." *NBER Macroeconomics Annual* 30 (1): 273–335.

Acemoglu, Daron, and Pablo D Azar. 2020. "Endogenous production networks." *Econometrica* 88 (1): 33–82.

Acemoglu, Daron, Vasco M Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2012. "The network origins of aggregate fluctuations." *Econometrica* 80 (5): 1977–2016.

Acemoglu, Daron, and Alireza Tahbaz-Salehi. 2020. "Firms, failures, and fluctuations: the macroeconomics of supply chain disruptions."Technical report, National Bureau of Economic Research.

Alessandria, George, Costas Arkolakis, and Kim J Ruhl. 2021. "Firm dynamics and trade." *Annual Review of Economics* 13 253–280.

Antras, Pol, and Davin Chor. 2021. "Global value chains."Technical report, National Bureau of Economic Research.

Atkeson, Andrew, Ariel T Burstein, and Manolis Chatzikonstantinou. 2019. "Transitional dynamics in aggregate models of innovative investment." *Annual Review of Economics* 11 273–301.

Ballester, Coralio, Antoni Calvó-Armengol, and Yves Zenou. 2006. "Who's who in networks. Wanted: The key player." *Econometrica* 74 (5): 1403–1417.

Baqaee, David, and Emmanuel Farhi. 2020. "Entry vs. Rents: Aggregation with Economies of Scale." *NBER Working Paper* (w27140): .

Baqaee, David Rezza. 2018. "Cascading failures in production networks." *Econometrica* 86 (5): 1819–1838.

Baqaee, David Rezza, and Emmanuel Farhi. 2018. "Macroeconomics with heterogeneous agents and input-output networks."Technical report, National Bureau of Economic Research.

Baqaee, David Rezza, and Emmanuel Farhi. 2019. "The macroeconomic impact of microeconomic shocks: Beyond Hulten's theorem." *Econometrica* 87 (4): 1155–1203.

Barrot, Jean-Noël, and Julien Sauvagnat. 2016. "Input specificity and the propagation of idiosyncratic shocks in production networks." *The Quarterly Journal of Economics* 131 (3): 1543–1592.

Bernard, Andrew B, Emmanuel Dhyne, Glenn Magerman, Kalina Manova, and Andreas Moxnes. 2022. "The origins of firm heterogeneity: A production network approach." *Journal of Political Economy* 130 (7): 000–000.

Bernard, Andrew B, and Andreas Moxnes. 2018. "Networks and trade."Technical report, National Bureau of Economic Research.

Bernard, Andrew B, Andreas Moxnes, and Yukiko U Saito. 2019. "Production networks, geography, and firm performance." *Journal of Political Economy* 127 (2): 639–688.

Bigio, Saki, and Jennifer La'o. 2020. "Distortions in production networks." *The Quarterly Journal of Economics* 135 (4): 2187–2253.

Boehm, Christoph E, Aaron Flaaen, and Nitya Pandalai-Nayar. 2019. "Input linkages and the transmission of shocks: Firm-level evidence from the 2011 Tōhoku earthquake." *Review of Economics and Statistics* 101 (1): 60–75.

Bonacich, Phillip. 1987. "Power and centrality: A family of measures." *American Journal of Sociology* 92 (5): 1170–1182.

Caballero, Ricardo J, Takeo Hoshi, and Anil K Kashyap. 2008. "Zombie lending and depressed restructuring in Japan." *American Economic Review* 98 (5): 1943–77.

Caliendo, Lorenzo, Fernando Parro, and Aleh Tsyvinski. 2022. "Distortions and the Structure of the World Economy." *American Economic Journal: Macroeconomics* 14 (4): 274–308.

Calvo, Guillermo A. 1983. "Staggered prices in a utility-maximizing framework." *Journal of Monetary Economics* 12 (3): 383–398.

Carvalho, Vasco M, Makoto Nirei, Yukiko U Saito, and Alireza Tahbaz-Salehi. 2021. "Supply chain disruptions: Evidence from the great east japan earthquake." *The Quarterly Journal of Economics* 136 (2): 1255–1321.

Carvalho, Vasco M, and Alireza Tahbaz-Salehi. 2019. "Production networks: A primer." *Annual Review of Economics* 11 635–663.

Dhingra, Swati, and John Morrow. 2019. "Monopolistic competition and optimum product diversity under firm heterogeneity." *Journal of Political Economy* 127 (1): 196–232.

Dhyne, Emmanuel, Ayumu Ken Kikkawa, Magne Mogstad, and Felix Tintelnot. 2021. "Trade and domestic production networks." *The Review of Economic Studies* 88 (2): 643–668.

Eaton, Jonathan, Samuel S Kortum, and Francis Kramarz. 2022. "Firm-to-Firm Trade: Imports, exports, and the labor market."Technical report, National Bureau of Economic Research.

Edmond, Chris, Virgiliu Midrigan, and Daniel Yi Xu. 2018. "How costly are markups?"Technical report, National Bureau of Economic Research.

Elliott, Matthew, and Benjamin Golub. 2022. "Networks and economic fragility." *Annual Review of Economics* 14 665–696.

Elliott, Matthew, Benjamin Golub, and Matthew V Leduc. 2022. "Supply network formation and fragility." *American Economic Review* 112 (8): 2701–47.

Grassi, Basile et al. 2017. "Io in io: Size, industrial organization, and the input-output network make a firm structurally important." *Working Papers*.

Hopenhayn, Hugo A. 2014. "Firms, misallocation, and aggregate productivity: A review." *Annual Review of Economics* 6 (1): 735–770.

Hosono, Kaoru, and Miho Takizawa. 2022. "Japan's productivity stagnation: Using dynamic Hsieh–Klenow decomposition." *Contemporary Economic Policy* 40 (1): 218–232.

Hsieh, Chang-Tai, and Peter J Klenow. 2009. "Misallocation and manufacturing TFP in China and India." *The Quarterly Journal of Economics* 124 (4): 1403–1448.

Huggett, Mark. 1996. "Wealth distribution in life-cycle economies." *Journal of Monetary Economics* 38 (3): 469–494.

Hulten, Charles R. 1978. "Growth accounting with intermediate inputs." *The Review of Economic Studies* 45 (3): 511–518.

Huneeus, Federico. 2020. "Production Network Dynamics and the Propagation of Shocks." *Working Papers*.

Imai, Kentaro. 2016. "A panel study of zombie SMEs in Japan: Identification, borrowing and investment behavior." *Journal of the Japanese and International Economies* 39 91–107.

Jones, Charles I. 2011. "Misallocation, economic growth, and input-output economics."Technical report, National Bureau of Economic Research.

König, Michael D, Andrei Levchenko, Tim Rogers, and Fabrizio Zilibotti. 2022. "Aggregate fluctuations in adaptive production networks." *Proceedings of the National Academy of Sciences* 119 (38): e2203730119.

Kopytov, Alexandr, Kristoffer Nimark, Bineet Mishra, and Mathieu Taschereau-Dumouchel. 2022. "Endogenous Production Networks under Supply Chain Uncertainty."

Kwon, Hyeog Ug, Futoshi Narita, and Machiko Narita. 2015. "Resource reallocation and zombie lending in Japan in the 1990s." *Review of Economic Dynamics* 18 (4): 709–732.

Leontief, Wassily W. 1941. "The Structure of American Economy, 1919–1929: An Empirical Application of Equilibrium Analysis."

Lim, Kevin. 2018. "Endogenous production networks and the business cycle." *Working Papers*.

Liu, Ernest. 2019. "Industrial policies in production networks." *The Quarterly Journal of Economics* 134 (4): 1883–1948.

Long Jr, John B, and Charles I Plosser. 1983. "Real business cycles." *Journal of political Economy* 91 (1): 39–69.

Lucas Jr, Robert E. 1990. "Supply-side economics: An analytical review." *Oxford Economic Papers* 42 (2): 293–316.

Melitz, Marc J. 2003. "The impact of trade on intra-industry reallocations and aggregate industry productivity." *Econometrica* 71 (6): 1695–1725.

Miyauchi, Yuhei. 2021. "Matching and agglomeration: Theory and evidence from japanese firm-to-firm trade." *Working Paper.*

Oberfield, Ezra. 2018. "A theory of input–output architecture." *Econometrica* 86 (2): 559–589.

OECD. 2016. *Japan: Boosting growth and well-being in an ageing society.* Paris: OECD iLibrary.

Polyanin, Andrei D, and Alexander V Manzhirov. 2008. *Handbook of integral equations.* Chapman and Hall/CRC.

Restuccia, Diego, and Richard Rogerson. 2017. "The causes and costs of misallocation." *Journal of Economic Perspectives* 31 (3): 151–74.

Stewart, Gilbert W. 1998. *Matrix algorithms: volume 1: basic decompositions.* SIAM.

Stokey, Nancy L. 1989. *Recursive methods in economic dynamics.* Harvard University Press.

Taschereau-Dumouchel, Mathieu. 2022. "Cascades and fluctuations in an economy with an endogenous production network." *Available at SSRN 3115854.*

Tintelnot, Felix, Ayumu Ken Kikkawa, Magne Mogstad, and Emmanuel Dhyne. 2018. "Trade and domestic production networks."Technical report, National Bureau of Economic Research.

Yamada, Kotone, Yukio Minoura, Jouchi Nakajima, and Tomoyuki Yagi. 2022. "[Corporate Financial Assistance and Resource Allocation: Research Trends and Trends after the Spread of COVID-19] Kigyou kinyuu shien to shigen haibun - kenkyu no choryu to shingata koronauirusu kansensyo kakudaigo nodoukou- (in Japanese)." *Bank of Japan Working Paper Series* (22-J-4): .

Yosida, K. 1980. "Functional Analysis, Classics in Mathematics Series." *Cambridge University Press* 66 68.

A Computation Algorithm

In this section, we explain a computation algorithm to obtain the fixed point in (2.21) and (2.22) and equilibria.

A.1 Upstream/Downstream Fixed Point

Though (2.21) and (2.22) are not analytically solvable, we can solve for $\Phi(\cdot)$ and $\Delta(\cdot)$ by iteration of the mappings because each of the system composes contraction mapping. The uniqueness and consistency are guaranteed by the contraction mapping theorem. For the detail of contraction mapping, check Stokey (1989).

0. Guess $\Phi(\chi)$.
1. By calculating integration in the right hand side given $\Phi(\chi)$, update $\Phi(\chi)$ and iterate it until convergence.

We can solve for $\Delta(\chi)$ in the same way, as discussed in 2.4.2.

A.2 Equilibria

B Proofs

In this section, We show the proofs for propositions.

B.1 Proof of Proposition 1 and 2

Using the discrete state space expression, (2.21) and (2.22) can be written as follows.

$$\boldsymbol{\Phi} = \boldsymbol{\phi}^{\sigma-1} + \beta_\Phi \boldsymbol{m}(\boldsymbol{M} \circ \boldsymbol{\Phi}) \tag{B.1}$$

$$\boldsymbol{\Delta} = \mu^{-\sigma}\boldsymbol{\delta}^{\sigma-1} + \beta_\Delta \boldsymbol{m}'(\boldsymbol{M} \circ \boldsymbol{\Delta}) \tag{B.2}$$

The i-th row of the equation (B.1) is

$$\Phi_i = \phi_i^{\sigma-1} + \beta_\Phi(m_{i1}M_1\Phi_1 + \cdots + m_{in}M_n\Phi_n). \tag{B.3}$$

Taking total derivative leads to

$$d\Phi_i = \beta_\Phi \left((m_{i1}M_1, \cdots, m_{in}M_n) \begin{pmatrix} d\Phi_1 \\ \vdots \\ d\Phi_n \end{pmatrix} + (m_{i1}, \cdots, m_{in}) \begin{pmatrix} dM_1\Phi_1 \\ \vdots \\ dM_n\Phi_n \end{pmatrix} \right) \tag{B.4}$$

$$= \beta_\Phi \left((m_{i1}M_1, \cdots, m_{in}M_n) \begin{pmatrix} d\Phi_1 \\ \vdots \\ d\Phi_n \end{pmatrix} + (m_{i1}M_1, \cdots, m_{in}M_n) \begin{pmatrix} \Phi_1 \dfrac{dM_1}{M_1} \\ \vdots \\ \Phi_n \dfrac{dM_n}{M_n} \end{pmatrix} \right). \tag{B.5}$$

Stacking them vertically for $i = 1, \cdots, n$ leads to

$$d\mathbf{\Phi} = \beta_\Phi \boldsymbol{N_s} d\mathbf{\Phi} + \beta_\Phi \boldsymbol{N_s} \left(\mathbf{\Phi} \circ \frac{d\boldsymbol{M}}{\boldsymbol{M}} \right). \tag{B.6}$$

This can be solved by

$$d\mathbf{\Phi} = (I - \beta_\Phi \boldsymbol{N_s})^{-1} \beta_\Phi \boldsymbol{N_s} \left(\mathbf{\Phi} \circ \frac{d\boldsymbol{M}}{\boldsymbol{M}} \right) \tag{B.7}$$

$$= (I + \beta_\Phi \boldsymbol{N_s} + \beta_\Phi^2 \boldsymbol{N_s}^2 + \cdots) \beta_\Phi \boldsymbol{N_s} \left(\mathbf{\Phi} \circ \frac{d\boldsymbol{M}}{\boldsymbol{M}} \right). \tag{B.8}$$

From (B.7) to (B.8), We used the condition that the spectral radius of $\beta_\Phi \boldsymbol{N_s}$ is less than 1 to derive the infinite sum of the powers of the network matrix $\boldsymbol{N}$ (Neumann series). This is verified in the same procedure as the discussion about the uniqueness of the fixed points in 2.4.2 using Blackwell's sufficient conditions. Check Stewart (1998) for more detail about the spectral radius of a matrix. The proof for Proposition 2 is the same.

□

B.2 Proof of Proposition 3

Taking log of (2.27) leads to

$$\log \Delta_H = \log(L - L^o) - \log(\boldsymbol{M'}(\boldsymbol{\phi}^{\sigma-1} \circ \Delta)). \tag{B.9}$$

Differentiating both sides leads to

$$\frac{d\Delta^H}{\Delta^H} = -\frac{dL^o}{L - L^o} - \frac{\sum_i (\phi_i^{\sigma-1} dM_i \Delta_i + \phi_i^{\sigma-1} M_i d\Delta_i)}{\boldsymbol{M'}(\boldsymbol{\phi}^{\sigma-1} \circ \boldsymbol{\Delta})} \tag{B.10}$$

$$= -w_o \frac{dL^o}{L^p} - \frac{\sum_i \left(L_i^p \left(\frac{dM_i}{M_i} + \frac{d\Delta_i}{\Delta_i} \right) \right)}{\Delta_H \boldsymbol{M'}(\boldsymbol{\phi}^{\sigma-1} \circ \boldsymbol{\Delta})} \tag{B.11}$$

$$= -w_o \frac{dL^o}{L^p} - \sum_i \left(w_i^L \left(\frac{dM_i}{M_i} + \frac{d\Delta_i}{\Delta_i} \right) \right) \tag{B.12}$$

$$\frac{d\Delta^H}{\Delta^H} = -w^o \frac{dL^o}{L^o} - \boldsymbol{w^{L'}} \left(\frac{d\boldsymbol{M}}{\boldsymbol{M}} + \frac{d\boldsymbol{\Delta}}{\boldsymbol{\Delta}} \right) \tag{B.13}$$

We used $\Delta^H = \dfrac{L^p}{\boldsymbol{M'}(\boldsymbol{\phi}^{\sigma-1} \circ \Delta)}$ (2.27) from (B.11) to (B.12).
Taking log of (2.29) leads to

$$\log P^H = \log \mu + \frac{1}{1-\sigma} \log(\boldsymbol{M'}(\boldsymbol{\delta}^{\sigma-1} \circ \mathbf{\Phi})). \tag{B.14}$$

Differentiating both sides leads to

$$\frac{dP^H}{P^H} = \frac{1}{1-\sigma}\frac{\sum_i(\delta_i^{\sigma-1}dM_i\Phi_i + \delta_i^{\sigma-1}M_i d\Phi_i)}{\boldsymbol{M}'(\boldsymbol{\delta}^{\sigma-1}\circ\boldsymbol{\Phi})} \tag{B.15}$$

$$= \frac{1}{1-\sigma}\frac{\sum_i(M_i\delta_i^{\sigma-1}\Phi_i\left(\frac{dM_i}{M_i}+\frac{d\Phi_i}{\Phi_i}\right)}{\boldsymbol{M}'(\boldsymbol{\delta}^{\sigma-1}\circ\boldsymbol{\Phi})} \tag{B.16}$$

$$= \frac{1}{1-\sigma}\sum_i\left(w_i^E\left(\frac{dM_i}{M_i}+\frac{d\Phi_i}{\Phi_i}\right)\right) \tag{B.17}$$

$$\frac{dP^H}{P^H} = \frac{1}{1-\sigma}\boldsymbol{w}^{\boldsymbol{E}'}\left(\frac{d\boldsymbol{M}}{\boldsymbol{M}}+\frac{d\boldsymbol{\Phi}}{\boldsymbol{\Phi}}\right) \tag{B.18}$$

and $E = \sum_i p_i x_i^H M_i = \sum_i \Delta_H \delta_i^{\sigma-1}(\mu\eta_i)^{1-\sigma}M_i$ from (B.16) to (B.17). Lastly, taking the log of (2.28) leads to

$$\log U = \log\Delta^H - \sigma\log P^H. \tag{B.19}$$

Taking derivative and substituting (B.13) and (B.18) derive (3.21).

□

C Analytical Solution for Integral Equations

In this section, we derive analytical solutions for two-way fixed point equations under some functional assumptions and derive some analytical explanations about the computational results. Both of the fixed point equations have the form so-called *Fredholm integral equations of the second kind*, which is used in various fields of science. There are several ways to obtain closed-form solutions or analyze the characteristics of the solution. For more detail, check Polyanin and Manzhirov (2008).

C.1 ϕ-economy

First, we analyze ϕ-economy where firms are heterogeneous with respect to their fundamental productivity, not to fundamental demand. Suppose the distributions of fundamental productivity and matching function follow the Pareto distribution as follows for $\phi \in [\underline{\phi}, \infty]$.

$$dM(\phi) = \left(\frac{\alpha_\phi}{\underline{\phi}}\right)\left(\frac{\phi}{\underline{\phi}}\right)^{-\alpha_\phi - 1} d\phi \tag{C.1}$$

$$m(\phi, \phi') = 1 - \left(\frac{\phi}{\underline{\phi}}\right)^{-\rho_\phi}\left(\frac{\phi'}{\underline{\phi}}\right)^{-\rho'_\phi} \tag{C.2}$$

All the newly defined parameters α_ϕ, ρ_ϕ, and ρ'_ϕ are in $\mathcal{R}^+$, which implies the mass of firms decreases in firm size and the matching function increases in the size of supplier and customer. Firstly, let's focus on the determination of productivity Φ in the backward fixed point equations (2.21).

$$\Phi(\phi) = \phi^{\sigma-1} + \alpha_\phi \beta_\Phi \int_{\underline{\phi}}^{\infty} \Phi(\phi')\left(\frac{\phi'}{\underline{\phi}}\right)^{-\alpha_\phi - 1} - \Phi(\phi')\left(\frac{\phi'}{\underline{\phi}}\right)^{-\alpha_\phi - 1}\left(\frac{\phi}{\underline{\phi}}\right)^{-\rho_\phi}\left(\frac{\phi'}{\underline{\phi}}\right)^{-\rho'_\phi}\frac{d\phi'}{\underline{\phi}} \tag{C.3}$$

Using change-of-variables as $\frac{\phi}{\underline{\phi}} = e^s (\Rightarrow \frac{1}{\underline{\phi}} d\phi = e^s ds)$, and defining a function $y(s) = \Phi(\underline{\phi} e^s)$, (C.3) becomes

$$y(s) = \gamma e^{ks} + \alpha_\phi \beta_\Phi \int_0^\infty y(s')e^{-\alpha_\phi s'} - y(s')e^{(-\alpha_\phi - \rho')s'}e^{-\rho s}ds' \tag{C.4}$$

where $\gamma = \left(\frac{1}{\underline{\phi}}\right)^{\sigma-1}$. Suppose $y(s) = C_0 + \gamma e^{ks} + C_1 e^{-\rho_\phi s}$ for some constant C_0 and $C_1 \in \mathcal{R}$. Then, under the assumption that the integration in the right-hand side does not explode, (i.e., $\alpha_\phi - k > 0$[37]) coefficient comparison leads to systems for C_0, C_1 as follows.

$$\frac{C_0}{\alpha_\phi \beta_\Phi} = C_0 \frac{1}{\alpha_\phi} + \gamma \frac{1}{\alpha_\phi - k} + C_1 \frac{1}{\rho + \alpha_\phi} \tag{C.5}$$

$$-\frac{C_1}{\alpha_\phi \beta_\Phi} = C_0 \frac{1}{\alpha_\phi + \rho'} + \gamma \frac{1}{\alpha_\phi + \rho' - k} + C_1 \frac{1}{\rho + \alpha_\phi + \rho'} \tag{C.6}$$

[37] Intuitively, this assumption requires the number of firms with extremely high productivity is not too high and productivity gain obtained by the network with these firms are not too high.

Hence, we obtain

$$\begin{pmatrix} C_0 \\ C_1 \end{pmatrix} = \frac{1}{|D|} \begin{pmatrix} \frac{1}{\alpha_\phi \beta} + \frac{1}{\alpha_\phi + \rho + \rho'} & \frac{1}{\alpha_\phi + \rho} \\ -\frac{1}{\alpha_\phi + \rho'} & \frac{1}{\alpha_\phi \beta} - \frac{1}{\alpha_\phi} \end{pmatrix} \begin{pmatrix} \gamma \frac{1}{\alpha - k} \\ -\gamma \frac{1}{\alpha + \rho' - k} \end{pmatrix} \tag{C.7}$$

where $|D| = \left(\frac{1}{\alpha_\phi \beta} - \frac{1}{\alpha_\phi}\right)\left(\frac{1}{\alpha_\phi \beta} + \frac{1}{\alpha_\phi + \rho + \rho'}\right) + \frac{1}{\alpha_\phi + \rho} \frac{1}{\alpha_\phi + \rho'} > 0$. Since $\beta_\Phi < 1$, we obtain $C_1 < 0$, which implies $y(s)$ is increasing in s not only from its fundamental productivity (γe^{ks}), but also from its network ($C_1 e^{-\rho_\phi s}$).
Nextly, let's focus on the determination of scaled demand $\Delta(\chi)$ in the forward fixed point equations (2.22).

$$\Delta(\phi) = c + \alpha_\phi \beta_\Delta \int_{\underline{\phi}}^{\infty} \Delta(\phi') \left(\frac{\phi'}{\underline{\phi}}\right)^{-\alpha_\phi - 1} - \Delta(\phi') \left(\frac{\phi'}{\underline{\phi}}\right)^{-\alpha_\phi - 1} \left(\frac{\phi'}{\underline{\phi}}\right)^{-\rho_\phi} \left(\frac{\phi}{\underline{\phi}}\right)^{-\rho'_\phi} \frac{d\phi'}{\underline{\phi}} \tag{C.8}$$

where $c = \mu^{-\sigma}\delta^{\sigma-1} > 0$ is constant and common across all the firms in ϕ-economy. In the same way with productivity, using change-of-variables as $\frac{\phi}{\underline{\phi}} = e^s (\Rightarrow \frac{1}{\underline{\phi}} d\phi = e^s ds)$, and defining a function $z(s) = \Delta(\underline{\phi} e^s)$, (C.8) becomes

$$z(s) = c + \alpha_\phi \beta_\Delta \int_0^\infty z(s') e^{-\alpha_\phi s'} - z(s') e^{(-\alpha_\phi - \rho)s'} e^{-\rho' s} ds' \tag{C.9}$$

Suppose $z(s) = C_0 + C_1 e^{-\rho'_\phi s}$ for some constant C_0 and $C_1 \in \mathcal{R}$. Then, coefficient comparison leads to systems for C_0, C_1 as follows.

$$C_0 = c + \alpha_\phi \beta_\Delta \left(C_0 \frac{1}{\alpha_\phi} + C_1 \frac{1}{\rho' + \alpha_\phi}\right) \tag{C.10}$$

$$C_1 = \alpha_\phi \beta_\Delta \left(C_0 \frac{1}{\alpha_\phi + \rho} + C_1 \frac{1}{\rho + \alpha_\phi + \rho'}\right) \tag{C.11}$$

Hence, we obtain

$$\begin{pmatrix} C_0 \\ C_1 \end{pmatrix} = \frac{1}{|D|} \begin{pmatrix} -\left(1 - \frac{\alpha_\phi \beta_\Delta}{\alpha_\phi + \rho + \rho'}\right) & \frac{\alpha_\phi \beta_\Delta}{\alpha_\phi + \rho'} \\ -\frac{1}{\alpha_\phi + \rho'} & 1 - \beta_\Delta \end{pmatrix} \begin{pmatrix} c \\ 0 \end{pmatrix} \tag{C.12}$$

where $|D| = (1 - \beta)\left(1 - \frac{\alpha_\phi \beta_\Delta}{\alpha_\phi + \rho + \rho'}\right) + \frac{\alpha_\phi \beta_\Delta}{\alpha_\phi + \rho} \frac{\alpha_\phi \beta_\Delta}{\alpha_\phi + \rho'} > 0$ from $\beta_\Delta < 1$. Note that $C_1 < 0$, which implies $z(s)$ is increasing in s. While fundamental demand is common across all the firms, scaled demand varies from its supplier network.
Comparing the results for productivity and scaled demand yields several consequences. Firstly, while the basis functions of productivity contain both e^{ks} and $e^{-\rho s}$, those of scaled demand

contain only $e^{-\rho' s}$ (except for scalar). This suggests that since e^{ks} has a positive second derivative unlike $e^{-\rho' s}$, productivity varies more widely than scaled demand does.[38] Secondly, remember that firm size is proportional to the product of productivity and scaled demand from (2.25). This suggests that in ϕ-economy, the difference in firm size is mainly driven by productivity. Finally, remember that profit from each network is a product of the supplier's productivity and the customer's scaled demand from (4.1). This suggests that small firms with small productivity have a low incentive to be a supplier, and they do not function as suppliers in ϕ-economy by endogenizing network formation. All the consequences above are consistent with the computational results shown in Figure 10, and in the δ-economy, the totally opposite holds true.

C.2 General Economy

In the same way with the case of ϕ-economy, we can solve for backward/forward fixed point equations in the general economy, i.e., both fundamental productivity and fundamental demand are heterogeneous under Pareto distributional assumption. Suppose the distribution of firm fundamentals and matching function follow the Pareto distribution as follows.

$$dM(\phi) = \phi^{-\alpha_\phi} d\phi \tag{C.13}$$

$$dM(\delta) = \delta^{-\alpha_\delta} d\delta \tag{C.14}$$

$$m(\phi, \phi') = 1 - \left(\frac{\phi}{\phi_*}\right)^{-\rho_\phi} \left(\frac{\phi'}{\phi_*}\right)^{-\rho'_\phi} \left(\frac{\delta}{\delta_*}\right)^{-\rho_\delta} \left(\frac{\delta'}{\delta_*}\right)^{-\rho'_\delta} \tag{C.15}$$

We focus on the determination of productivity Φ in the backward fixed point equations (2.21).[39] Using change-of-variables as $\phi = e^s (\Rightarrow \log(\phi) = s, d\phi = e^s ds)$ and $\delta = e^t$, and defining a function $y(s,t) = \Phi(e^s, e^t) (\Rightarrow y(\log(\phi), \log(\delta)) = \Phi(\phi, \delta))$, (2.21) becomes

$$\begin{aligned} y(s,t) = e^{ks} &+ \beta \int_{\log(\underline{\delta})}^{\infty} \int_{\log(\underline{\phi})}^{\infty} y(s',t') e^{-\alpha^*_\phi s'} e^{-\alpha^*_\delta t'} ds' dt' \\ &- e^{\rho_\phi s + \rho_\delta t} \frac{\beta}{A} \int_{\log(\underline{\delta})}^{\infty} \int_{\log(\underline{\phi})}^{\infty} y(s',t') e^{(-\alpha^*_\phi + \rho'_\phi)s'} e^{(-\alpha^*_\delta + \rho'_\delta)t'} ds' dt' \end{aligned} \tag{C.16}$$

where $k = \sigma - 1, -\alpha^*_\phi = 1 - \alpha_\phi, -\alpha^*_\delta = 1 - \alpha_\delta$, and $A = A_\phi A_\delta = \underline{\phi}^{-(\rho_\phi + \rho'_\phi)} \underline{\delta}^{-(\rho_\delta + \rho'_\delta)}$.
Here, suppose $y(s,t) = C_0 + e^{ks} + C_1 e^{\rho_\phi s + \rho_\delta t}$ for some constant $C_0, C_1 \in \mathcal{R}$. Then, we obtain systems for C_0, C_1 by a coefficient comparison method as follows.

$$\frac{C_0}{\beta} = \frac{\underline{\delta}^{-\alpha^*_\delta}}{\alpha^*_\delta} \left(C_0 \frac{\underline{\phi}^{-\alpha^*_\phi}}{\alpha^*_\phi} + \frac{\underline{\phi}^{-\alpha^*_\phi + k}}{-\alpha^*_\phi + k} \right) + C_1 \frac{\underline{\phi}^{-\alpha^*_\phi + \rho_\phi}}{-\alpha^*_\phi + \rho_\phi} \frac{\underline{\delta}^{-\alpha^*_\delta + \rho_\delta}}{-\alpha^*_\delta + \rho_\delta} \tag{C.17}$$

$$-\frac{C_1}{\beta/A} = \frac{\underline{\delta}^{(-\alpha^*_\delta + \rho'_\delta)}}{-\alpha^*_\delta + \rho'_\delta} \left(C_0 \frac{\underline{\phi}^{(-\alpha^*_\phi + \rho'_\phi)}}{(-\alpha^*_\phi + \rho'_\phi)} + \frac{\underline{\phi}^{(-\alpha^*_\delta + \rho'_\delta) + k}}{(-\alpha^*_\delta + \rho'_\delta) + k} \right) + C_1 \frac{\underline{\phi}^{\alpha_\phi + (-\alpha^*_\phi + \rho'_\phi)}}{\alpha_\phi + (-\alpha^*_\phi + \rho'_\phi)} \frac{\underline{\delta}^{(-\alpha^*_\delta + \rho'_\delta)}}{\alpha_\delta + (-\alpha^*_\delta + \rho'_\delta)} \tag{C.18}$$

[38] Strictly speaking, if the cumulative distribution function on fundamental productivity diminishes too fast, it is possible that the variance of productivity is smaller than that of scaled demand.

[39] Just exchanging subscripts between ϕ and δ leads to the solutions for upward fixed point equations.

Solving the above leads to a solution for

$$\text{RHS} = e^{ks} + \beta e^{ax} \int_{\log(\underline{\phi})}^{\infty} \left(e^{kx'} + C e^{ax'} \right) e^{(1+b-d)x'} dx' \tag{C.19}$$

$$= e^{kx} + \beta e^{ax} \left[\frac{e^{(k+1+b-d)x'}}{k+1+b-d} + C \frac{e^{(a+1+b-d)x'}}{a+1+b-d} \right]_{\log \underline{\phi}}^{\infty} \tag{C.20}$$

D Relationships between β and σ

For $\sigma > 1$, we obtain

$$\frac{d}{d\sigma}\beta_{\Phi} = \frac{d}{d\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} = \frac{\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(1-\sigma\log\left(\frac{\sigma}{\sigma-1}\right)\right)}{\sigma} < 0 \tag{D.1}$$

$$\frac{d}{d\sigma}\beta_{\Delta} = \frac{d}{d\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} = \frac{\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(1-(\sigma-1)\log\left(\frac{\sigma}{\sigma-1}\right)\right)}{\sigma} > 0. \tag{D.2}$$

For β_{Φ}, given an increase in σ, the effect of an increase in elasticity surpasses the effect of the decrease in markup, and downstream propagation coefficient β_{Φ} is decreasing in σ. For β_{Δ}, the effect of a decrease in markup surpasses the effect of the increase in elasticity, and the upstream propagation coefficient β_{Δ} is increasing in σ. Figure 16 shows the relationship between β and σ for $\sigma > 1$. The difference arises from the difference in each power term to markup.

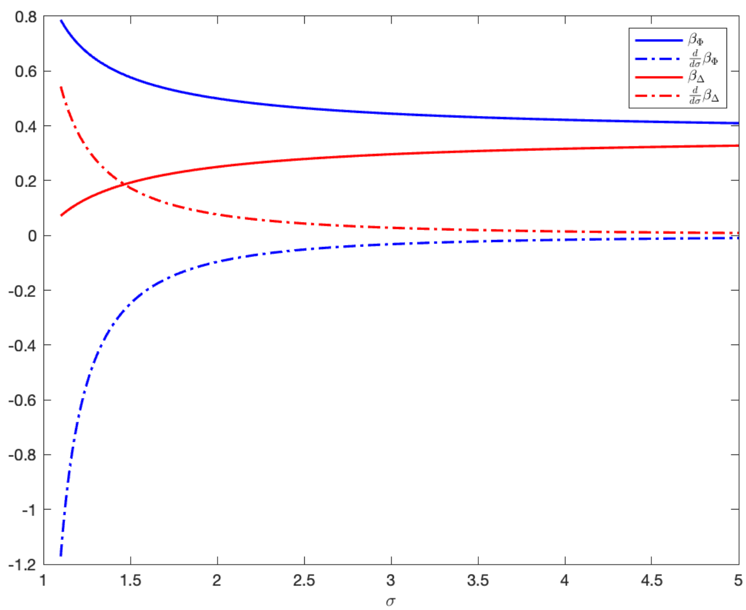


Figure 16: Relationships between β and σ

E Model Extension

E.1 Dynamic Model with Matching Friction

In this subsection, we extend the baseline model into a dynamic one. This includes dynamic decisions of firms' exit and of network formation with matching friction.
Time is discrete, and for a generic variable x in the main article, x_t represents its value at time t without otherwise being mentioned.

E.1.1 Household

We define the discount factor β for household lifetime utility just to characterize firms' discount rates. The household's lifetime utility is given by

$$V_t = \sum_{s=t}^{\infty} \beta^{s-t} U_s. \tag{E.1}$$

For simplicity, we assume there is no way to save, and intratemporal optimization of the household is identical in the baseline model.

E.1.2 Firm's Exit

In this dynamic model, we assume the firm fundamental does not change after entry like Melitz (2003). We characterize a firm by $(\phi, \delta) = \chi$ and its age a. The age a of (χ, a)-firm denotes how many periods have passed after the firm enters.
There are three possibilities for a firm's exit. First, (χ, a)-firm endogenously exits if $V_t(\chi, a) < 0$. Second, it suddenly dies with probability γ. Third, it must die if its terminal age $a = T_a$ is reached.[40] Due to the existence of a terminal age, the value function can be solved backward from terminal age T_a like Huggett (1996).[41] Then, $(\chi, a < T_a)$-firm's value function at t after realization of operation cost ϵ is defined as

$$V_t(\chi, a|\epsilon) = \max\left\{0, \pi_t(\chi, a) - \epsilon + \beta\gamma E_t\left[-l_{t+1}^f(\chi, a) + V_{t+1}(\chi, a+1|\epsilon')\right]\right\} \tag{E.2}$$

Since firms with age T_a has to exit without producing at the next period, the terminal value can be written by

$$V_{t+T_a-a}(\chi, T_a|\epsilon) = \max\{0, \pi_{t+T_a-a}(\chi, T_a) - \epsilon\} \tag{E.3}$$

$$= \begin{cases} \pi_{t+T_a-a}(\chi, T_a) - \epsilon & \text{if } \epsilon < \pi_{t+T_a-a}(\chi, T_a) \\ 0 & \text{otherwise.} \end{cases} \tag{E.4}$$

[40] By setting T_a large enough compared to the stochastic death rate γ, we can ignore its effects on GE.

[41] Even without setting terminal age or by setting $T_a \to \infty$, we can well-define equilibrium of this economy. However, to solve the model quantitatively, the infinite horizon model requires an iterative scheme to derive the policy function and the associated value function on the state space of the value function. The problem is that state space is an infinite dimension because of the stickiness of a network that consists of infinite-dimensional state space. As in this model, it's possible to include a as a state space instead of the network. Even so, state space on age is infinite as long as there is no terminal age.

and its expected value is given by

$$E_t\left[V_{t+T_a-a}(\chi,T_a|\epsilon)\right] = Pr\left(\epsilon<\pi_{t+T_a-a}(\chi,T_a)\right)E\left[\pi_{t+T_a-a}(\chi,T_a)-\epsilon|\epsilon<\pi_{t+T_a-a}(\chi,T_a)\right] \quad (E.5)$$
$$= G_\epsilon(\pi_{t+T_a-a}(\chi,T_a))\left(\pi_{t+T_a-a}(\chi,T_a)-c_\epsilon(\pi_{t+T_a-a}(\chi,T_a))\right) \quad (E.6)$$

In a similar way, we obtain the value function and its expected value for firms with $0<\tau<T_a-a$ as follows.

$$V_{t+\tau}(a+\tau|\epsilon) = \max\left\{0, \pi_{t+\tau}(\chi,a+\tau)-\epsilon+\beta\gamma E\left[-l^f_{t+\tau+1}(\chi,a+\tau+1)+V_{t+\tau+1}(\chi,a+\tau+1|\epsilon)\right]\right\}$$
$$= \begin{cases} \pi_{t+\tau}(\chi,a+\tau)-\epsilon+\beta\gamma E\left[-l^f_{t+\tau+1}(\chi,a+\tau+1)+V_{t+\tau+1}(\chi,a+\tau+1|\epsilon')\right] \\ \qquad \text{if } \epsilon<\pi_{t+\tau}(\chi,a+\tau)+\beta\gamma E\left[-l^f_{t+\tau+1}(\chi,a+\tau+1)+V_{t+\tau+1}(\chi,a+\tau+1|\epsilon')\right] \\ 0 \quad \text{otherwise} \end{cases} \quad (E.7)$$

$$E\left[-l^f_{t+\tau}(\chi,a+\tau)+V_{t+\tau}(\chi,a+\tau|\epsilon')\right]$$
$$= -l^f_{t+\tau}(\chi,a+\tau)+G_\epsilon\left(\pi_{t+\tau}(\chi,a+\tau)+\beta\gamma E\left[-l^f_{t+\tau+1}(\chi,a+\tau+1)+V_{t+\tau+1}(\chi,a+\tau+1|\epsilon')\right]\right)$$
$$\times\left(\pi_{t+\tau}(\chi,a+\tau)-c_\epsilon\left(\pi_{t+\tau}(\chi,a+\tau)+\beta\gamma E\left[-l^f_{t+\tau+1}(\chi,a+\tau+1)+V_{t+\tau+1}(\chi,a+\tau+1|\epsilon')\right]\right)\right.$$
$$\left.+\beta\gamma E\left[-l^f_{t+\tau}(\chi,a+\tau)+V_{t+\tau+1}(\chi,a+\tau+1|\epsilon')\right]\right) \quad (E.8)$$

From terminal value (E.4), (E.6) and from backward iteration (E.8), we can calculate the value function of firms. Using the value, we can define endogenous survival rate $h_t(\chi,a)$ by

$$h_t(\chi,a) = Pr\left(\epsilon\le\pi_t(\chi,a)+\beta\gamma E_t\left[-l^f_{t+1}(\chi,a+1)+V_{t+1}(\chi,a+1|\epsilon')\right]\right) \quad (E.9)$$
$$= G_\epsilon\left(\pi(\chi,a)+\beta\gamma E_t\left[-l^f_{t+1}(\chi,a+1)+V_{t+1}(\chi,a+1|\epsilon')\right]\right) \quad (E.10)$$

and the resulting distribution of firms is as follows.

$$M_t(\chi,a) = \begin{cases} h_t(\chi,a=1)M^e_tG_\chi(\chi) & \text{if } a=1 \\ \gamma h_t(\chi,a)M_{t-1}(\chi,a-1) & \text{if } 1<a\le T_a \end{cases} \quad (E.11)$$

Here, firms exit either by endogenous decision, reaching terminal age, or exogenous death shock which occurs with probability γ, independently across firms and their history.

E.1.3 Environment of Network Formation

Here, we explain dynamic network formation decisions based on Huneeus (2020). In addition to relationship cost shock as in the static model, we assume there is *reset shock* which determines whether the potential relationship can be reset at the period. All the potential networks get reset shock with probability $1-\nu$ independently from their history.[42] For each potential network, if a

[42] This takes a similar structure to Calvo (1983). In his model, though each firm's price differs due to whether the price reset shock occurs or not, the aggregate price index is determined exactly due to the law of large numbers. Here, the law of large numbers is applied to each supplier because there is a continuum of potential customers and all the firms with the same characteristic (χ,a,χ',a') have the same network.

reset shock does not arrive, the network status (the link exists/not) cannot be changed. Once reset shock arrives, the relationship cost is realized, and the supplier makes an activation decision comparing the lifetime profit via the network and the realized relationship cost.

E.1.4 Network Formation Decision

Suppose (χ', a')-firm tries to be a supplier for (χ, a)-firm. To keep the following equations simple, we define a two-way survival function as $H_t(\chi, a, \chi', a') = h_t(\chi, a)h_t(\chi, a')$, which returns the probability that both supplier and customer survive from operation cost realization at period t. Additionally, we define $d_t(\chi, a) = \mathbb{1}_{a \le T_a}(\chi, a)$ and $\mathbb{H}_t(\chi, a, \chi', a') = d_t(\chi, a)d_t(\chi', a')H_t(\chi, a, \chi', a')$ which takes deterministic death information into consideration, and if $a > T_a$ or $a' > T_a$, it returns 0 because they must die at $a = T_a$.
Then, the lifetime value of activating a relationship $V_t^+(\chi, a, \chi', a'|\xi)$ and that of not $V_t^-(\chi, a, \chi', a'|\xi)$ conditional on the realization of reset shock and relationship cost ξ can be expressed respectively as,

$$V_t^+(\chi, a, \chi', a'|\xi) = H_t(\chi, a, \chi', a')(\pi_t(\chi, a, \chi', a') - \xi) + \mathbb{H}_{t+1}(\chi, a+1, \chi', a'+1)\beta\gamma\left[\nu E_t\left[V_{t+1}^+(\chi, a+1, \chi', a'+1)\right] + (1-\nu)E_t\left[V_{t+1}^o(\chi, a+1, \chi', a'+1)\right]\right]. \tag{E.12}$$

$$V_t^-(\chi, a, \chi', a'|\xi) = \mathbb{H}_{t+1}(\chi, a+1, \chi', a'+1)\beta\gamma\left[\nu E_t\left[V_{t+1}^+(\chi, a+1, \chi', a'+1)\right] + (1-\nu)E_t\left[V_{t+1}^o(\chi, a+1, \chi', a'+1)\right]\right]. \tag{E.13}$$

where $V_t^o(\chi, a, \chi', a'|\xi)$ denotes the value to a (χ', a')-firm of having the option to reset its relationship with a (χ, a)-firm given the relationship cost shock ξ and expressed in

$$V_t^o(\chi, a, \chi', a'|\xi) = \max_{\{\text{activation,not}\}}\left\{V_t^+(\chi, a, \chi', a'|\xi), V_t^-(\chi, a, \chi', a'|\xi)\right\}. \tag{E.14}$$

Take the difference between the above, we obtain

$$V_t^+(\chi, a, \chi', a'|\xi) - V_t^-(\chi, a, \chi', a'|\xi) = H_t(\chi, a, \chi', a')(\pi_t(\chi, a, \chi', a') - \xi) + \mathbb{H}_{t+1}(\chi, a+1, \chi', a'+1)\beta\gamma\nu E_t\left[V_{t+1}^+(\chi, a+1, \chi', a'+1) - V_{t+1}^-(\chi, a+1, \chi', a'+1)\right] \tag{E.15}$$

Iterating forward until the (χ, a)-supplier must die at its terminal age, [43] we obtain

$$V_t(\chi, a, \chi', a'|\xi) = V_t^+(\chi, a, \chi', a'|\xi) - V_t^-(\chi, a, \chi', a'|\xi) \tag{E.16}$$

$$= H_t(\chi, a, \chi', a')(\pi_t(\chi, a, \chi', a') - \xi) + E_t\left[\sum_{s=1}^{a^T - a}\left(\prod_{\tau=1}^{s}\mathbb{H}_{t+\tau}(\chi, a+\tau, \chi', a'+\tau)\right)(\beta\gamma\nu)^s\left[\pi_{t+s}(\chi, a+s, \chi', a'+s)\right]\right]. \tag{E.17}$$

[43] The death of the potential customer by its terminal age is included in $\mathbb{H}_t(\chi, a, \chi', a')$

Since there are infinitely many customers on (χ, a), the acceptance function which returns the probability that *activation* is chosen in the maximization problem of option price (E.14) can be written as

$$a_t(\chi, a, \chi', a') = Pr\left[V_t^+(\chi, a, \chi', a') > V_t^-(\chi, a, \chi', a')\right] \tag{E.18}$$

$$= F_\xi\left(H_t(\chi, a, \chi', a')\,\pi_t(\chi, a, \chi', a') + \sum_{s=1}^{a^T - a}\left(\prod_{\tau=1}^{s}\mathbb{H}_{t+\tau}(\chi, a+\tau, \chi', a'+\tau)\right)(\beta\gamma\nu)^s\left[\pi_{t+s}(\chi, a+s, \chi', a'+s)\right]\right) \tag{E.19}$$

$$= F_\xi(V(\chi, a, \chi', a')) \tag{E.20}$$

Since $1-\nu$ fraction of the network is not endowed with reset shock, they remain unchanged from the last period. Hence, given the matching function of the last period, the update on the entire matching function can be written as follows.

$$m_t(\chi, a, \chi', a') = \begin{cases} \nu a_t(\chi, a, \chi', a')\text{ (if } a = 0 \vee a' = 0) \\ (1-\nu)m_{t-1}(\chi, a-1, \chi, a'-1') + \nu a_t(\chi, a, \chi', a') \\ \quad \text{(if } a > 0 \wedge a' > 0) \end{cases} \tag{E.21}$$

Note that entrants have no network before entry, and $m_t(\chi, 0, \chi', a') = m_t(\chi, a, \chi', 0) = 0$.
Given the decision-making about dynamic network formation above, realizing the relationship cost actually paid by supplier (χ', a')-firm is

$$l_t^f(\chi', a') = \int (1-\nu)\bar{\xi}\left(V_t(\chi, a, \chi', a')\right) d\left(\frac{M_t(\chi, a)}{h_t(\chi, a)}\right) \tag{E.22}$$

E.1.5 Equilibrium

We define the concept of dynamic equilibrium under endogenous network decisions as follows. (For brevity, we denote $\boldsymbol{\chi} = (\chi, a)$.)

Definition 3 (Dynamic Equilibrium)**.** Given an initial matching function $m_0(\cdot)$ and distribution function $F_0(\cdot)$, dynamic market equilibrium is a sequence of allocation functions $\{\Delta_t(\boldsymbol{\chi}), \Delta_t\}_{t=1}^{\infty}$, price functions $\{\Phi_t(\boldsymbol{\chi})\}_{t=1}^{\infty}$, a distribution function $\{M_t(\boldsymbol{\chi})\}_{t=1}^{\infty}$, and a matching function $\{m_t(\boldsymbol{\chi}, \boldsymbol{\chi})\}_{t=1}^{\infty}$ such that (i) consumer chooses consumption to maximizes lifetime utility, (ii) firms choose production input, price, network and make a stay/exit decision to maximize its lifetime value (iii) the free-entry condition is satisfied, (iv) all markets clear, and (v) the distribution follows the law of motion for each $t = \{1, \cdots, \infty\}$.

E.1.6 Computation Algorithm

Lastly, we show a computation algorithm to solve the dynamic model. The basic strategy is the same as Huneeus (2020), but to endogenize firms' exit decisions, we incorporate a step for calculating firms' value function backward like Huggett (1996).
First, solve for two steady states for $t = 0$ (head steady state) and $t = T+1$ (tail steady state). Given the steady-state value at $t = 0$ and $t = T+1$, iterate the below.

0. Guess $\{\pi_t(\boldsymbol{\chi}), \pi_t(\boldsymbol{\chi}, \boldsymbol{\chi}')\}_{t=1,\ldots,T}$.

1. Calculate value function $\{V_t(\boldsymbol{\chi})\}_{t=1,\ldots,T}$ backward from $t = T+1$ and associate survival function $\{h_t(\boldsymbol{\chi})\}_{t=1,\ldots,T}$.
2. Calculate distribution function $M_t(\boldsymbol{\chi})$ forward from $t = 0$.
3. Calculate value function $V_t(\boldsymbol{\chi}, \boldsymbol{\chi}')$ backward from $t = T+1$.
4. Calculate matching function $m_t(\boldsymbol{\chi}, \boldsymbol{\chi}')$ forward from $t = 0$.
5. Given $M_t(\boldsymbol{\chi})$ and $m_t(\boldsymbol{\chi}, \boldsymbol{\chi}')$, solve for $\{\Phi_t(\boldsymbol{\chi}), \Delta_t(\boldsymbol{\chi})\}_{t=1,\ldots,T}$
6. Calculate $\{\Delta_t^H\}_{t=1,\ldots,T}$.
7. Update $\{\pi_t(\boldsymbol{\chi}), \pi_t((\boldsymbol{\chi}, \boldsymbol{\chi}'))\}_{t=1,\ldots,T}$ and back to 1. until it converges.

If $\pi_T(\boldsymbol{\chi})$ or $\pi_T(\boldsymbol{\chi}, \boldsymbol{\chi}')$ is far from its tail steady state value $\pi_{T+1}(\boldsymbol{\chi}), \pi_{T+1}(\boldsymbol{\chi}, \boldsymbol{\chi}')$, increase T.

F Tables

Here, we show some tables which complement the main article.

F.1 Parameters

Table 1 is a summary of parameter value and its determination.

Parameter	Description	Value (ϕ-economy)	Value (δ-economy)	Target
σ	Elasticity of substitution	3		Hsieh and Klenow (2009)
L	General labor supply	0.93		Earlier ver. of Lim (2018)
L^f	Network labor supply	0.07		
M_e	Mass of entrants	2		Assumption
ϵ	Operation cost	$\epsilon \underset{i.i.d.}{\sim} \mathcal{N}(0.15, 0.05)$		Occupation share in Japan (35%)
ϕ	Fundamental productivity	$\log(\phi) \underset{i.i.d.}{\sim} \mathcal{N}(0, 1)$	$\log(\phi) = 0$	FSD in Japan
δ	Fundamental demand	$\log(\delta) = 0$	$\log(\delta) \underset{i.i.d.}{\sim} \mathcal{N}(0, 0.85)$	(The mean at the $\eta_{.80}$)
ξ	Network cost	$\xi \underset{i.i.d.}{\sim} U[0, 0.075]$		Final goods share in Japan (64%)
s_0	subsidy before reform	0.6	0.9	Business exit rate in Japan (0.05)
s_1	subsidy after reform	0.45	0.75	Business exit rate in the U.S. (0.1)

Table 1: Calibrated Parameters

F.2 Specific Value of Transition

We show the tables of specific values plotted in Section 5.

	$t = 0$	$t = 1$	$t = 2$
Mass of firms	1.895	1.747	1.784
U	3.850	3.778	3.914
L^p	0.646	0.668	0.662
L^o	0.284	0.262	0.268
Δ^H	0.151	0.163	0.157
P^H	0.340	0.351	0.343
w^f	1.932	-	1.833
Mass of networks	0.017	0.015	0.016

Table 2: Dynamics of aggregate variables in ϕ-economy (real value)

	$t=0$	$t=1$	$t=2$
Mass of firms	0.000	-0.078	-0.058
U	0.000	-0.019	0.017
L^p	0.000	0.034	0.026
L^o	0.000	-0.078	-0.058
Δ^H	0.000	0.078	0.041
P^H	0.000	0.032	0.008
w^f	0.000	-	-0.051
Mass of networks	0.000	-0.114	-0.047

Table 3: Dynamics of aggregate variables in ϕ-economy (rate of change)

	$t=1$	$t=2$
Reallocation from operation	0.034	0.026
Reallocation from production	0.004	0.001
Network reallocation	0.036	0.014
Love of variety (Change in P^H)	-0.064	-0.044
Productivity (Change in P^H)	-0.029	0.014

Table 4: Decomposition of change in welfare in ϕ-economy (rate of change)

	$t=0$	$t=1$	$t=2$
Mass of firms	1.904	1.796	1.810
U	3.329	3.293	3.358
L^p	0.644	0.661	0.659
L^o	0.286	0.269	0.271
Δ^H	0.204	0.216	0.212
P^H	0.394	0.403	0.398
w^f	1.609	-	1.529
Mass of networks	0.018	0.016	0.017

Table 5: Dynamics of aggregate variables in δ-economy (real value)

	$t=0$	$t=1$	$t=2$
Mass of firms	0.000	-0.057	-0.049
U	0.000	-0.011	0.009
L^p	0.000	0.025	0.022
L^o	0.000	-0.057	-0.049
Δ^H	0.000	0.055	0.039
P^H	0.000	0.022	0.010
w^f	0.000	-	-0.050
Mass of networks	0.000	-0.075	-0.050

Table 6: Dynamics of aggregate variables in δ-economy (rate of change)

	$t = 1$	$t = 2$
Reallocation from operation	0.025	-0.003
Reallocation from production	0.024	-0.004
Network reallocation	0.005	-0.008
Love of variety	-0.005	0.002
Productivity gain	-0.058	0.032

Table 7: Decomposition of change in welfare in δ-economy (rate of change)